Link Disutility Function for Risk Averse Drivers in a Stochastic Network

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BACKGROUND

At present, most traffic assignment models used in practice assume that travel time in a link is a deterministic function of the link's volume. A network with such a deterministic link travel time function is called a Deterministic Network (DN) (*1*). In reality, for the same volume in a link, we have variations in travel time. A network with such variation in link travel times may be called a Stochastic Network (SN) (*1*).

In a SN, driver's route choice in response to travel time uncertainty has been modeled (*2, 3, 4*). Instead of selecting the route which has the minimum expected travel time, the driver is modeled to select the route that has the minimum expected disutility. Under certain conditions, the traffic assignment problem in a SN can be solved by the well-known Deterministic User Equilibrium (DUE) algorithms simply by replacing the deterministic link travel time function with a suitable equivalent link disutility (ELD) function (*2, 3, 5*).

To assign traffic for the peak hour commute in a SN, a simple ELD function will be presented in this paper. This function takes into account the link characteristics and driver's response to uncertainty in link travel time, but unlike in (*2, 3, 5*), it does not require the modeler to specify the link travel time distribution or variance.

REVIOUS RELATED WORK

The Bureau of Public Road (BPR) function is the most popular function that describes the deterministic link travel time t_i in link *i* in a DN:

$$t_i = t_i^f \left[1 + \alpha \left(\frac{v_i}{c_i} \right)^{\beta} \right] \tag{1}$$

where t_i^f is the free-flow travel time in link *i*, v_i is the volume in link *i*, c_i is the capacity of link *i*, and α and β are constants. Typical values of α and β are 0.15 and 4 respectively.

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To model a SN, the authors of (3, 5) modeled t_i as Gamma distribution with a lower bound equal to t_i^f . In a SN, it is reasonable to assume that (1) describes the average link travel time $\bar{t}_i$ in link *i*:

$$\bar{t}_i = t_i^f \left[1 + \alpha \left(\frac{v_i}{c_i} \right)^\beta \right] \tag{2}$$

In a SN, since link travel times are stochastic, the route travel times are stochastic. The driver's route selection depends on how the he/she react to the route travel time uncertainty. The risk averse behavior is commonly used to describe drivers in the morning commute. The term risk here refers to the risk of a late arrival at the destination. A risk averse driver prefers a route that has a longer average travel time but smaller variance to a route that has a faster average travel time but higher variance. That is, he/she would avoid the routes that have high travel time variance to lower the risk of arriving late.

For the risk averse drivers the ELD function takes the following form (5):

$$DU_i = \bar{t}_i + c\sigma_{t_i}^2 \left(\frac{1}{2} + \frac{1}{3} c \frac{\sigma_{t_i}^2}{\bar{t}_i - t_i^f} \right) \tag{3}$$

where $\sigma_{t_i}^2$ is the variance of the travel time of link *i* and *c* is a constant determined by the parameters of the exponential disutility function. It is important to note that, in order to use (3), one needs to know the $\sigma_{t_i}^2$ for every link.

DERIVATION OF SIMPLE EQUIVALENT LINK DISUTILITY FUNCTION

Assume that DU_i depends on t_i^f and the relative average delay *d*, i.e.,

$$DU_i = F\left(t_i^f, d\right) \tag{4}$$

where

$$d = \frac{\bar{t}_i - t_i^f}{t_i^f} = \alpha \left(\frac{v_i}{c_i} \right)^\beta \tag{5}$$

for some function $F\left(t_i^f, d\right)$. $F\left(t_i^f, d\right)$ must be an increasing function of t_i^f . As the link becomes more congested, the variation of t_i and therefore the link disutility would increase; so, $F\left(t_i^f, d\right)$ must also be an increasing function of *d*. The function $F\left(t_i^f, d\right)$ must also satisfy the following conditions:

(i) When $v_i = 0$, $d = 0$, $t_i = \bar{t}_i = t_i^f$, therefore

$$F\left(t_i^f, 0\right) = t_i^f \tag{6}$$

(ii) For different *d*, we may write

$$DU_i = F\left(t_i^f, d\right) = t_i^f k(d) \tag{7}$$

For typical values of α and β, from (5) we have $d << 1$. Expanding $k(d)$ into a Taylor series, and ignoring the higher order terms, we obtain

$$DU_i = t_i^f \left[1 + a_1 \alpha \left(\frac{v_i}{c_i} \right)^{\beta} + a_2 \alpha^2 \left(\frac{v_i}{c_i} \right)^{2\beta} \right] \tag{8}$$

Furthermore, for the standard values of α =0.15, β =4, and the normal range of v_i / c_i, the term $\alpha^2 (v_i / c_i)^{2\beta}$ is usually negligible. Therefore we may simplify (8) as

$$DU_i = t_i^f \left[1 + a_1 \alpha \left(\frac{v_i}{c_i} \right)^{\beta} \right] = \bar{t}_i + t_i^f \left[(a_1 - 1) \alpha \left(\frac{v_i}{c_i} \right)^{\beta} \right] \tag{9}$$

Hence, we may view DU_i as consisting of two components: the "deterministic" component $\bar{t}_i$ which has the same value given by the BPR function, and the "stochastic" component $t_i^f [...]$ which is due to the uncertainty in link travel time. Then, a_1 describes the sensitivity of the driver in respond to this uncertainty. We called a_1 risk averse coefficient in this paper. For risk averse drivers, we can show that $a_1 \geq 1$ *(6)*. Note that, when a_1=1, drivers do not consider travel time uncertainty in route choice, and (9) is reduced to the BPR function.

Since a route consists of a series of connected links, the route's disutility may be computed by summing all the ELD values calculated in (9) from all the links. It has been shown that the sum of all the ELD values from all the links in a route leads to consistent route choice decisions of the drivers *(7)*.

To use (9) in a DUE algorithm, one only needs to know the value of a_1. In principle, every driver should have his/her individual a_1 value. To describe the general behavior of the driving population, average value of a_1 may be used.

ESTIMATION OF RISK AVERSE COEFFICIENT

By expressing (9) in terms of d, we obtain

$$DU_i = t_i^f [1 + a_1 d] = t_i^f \left[1 + a_1 \left(\frac{\bar{t}_i - t_i^f}{t_i^f} \right) \right] \tag{10}$$

Suppose that there are only two parallel links connecting an O-D pair. Equating the ELD of a link *i*=1 that has a constant travel time with link *i*=2 that has a stochastic travel time

$$t_1^f = t_2^f \left[1 + a_1 \left(\frac{\bar{t}_2 - t_2^f}{t_2^f} \right) \right] \tag{11}$$

We may then solve for a_1.

A questionnaire survey has been conducted in the city of El Paso, Texas, to estimate the average a_1 value among the driving population. In this survey, participants were presented with the scenario of morning commute to work that has a fixed work-start time with a penalty for late arrival. There are two questions in the survey. Question 1 has t_2^f=20 minutes and $\bar{t}_2$=30 minutes while Question 2 has t_2^f=35 minutes and $\bar{t}_2$=50. In each of the questions, participants were given a set of possible t_1^f values at 5-minute increments. Each person was asked to select the closest t_1^f value in each question that satisfies (11), that is, he/she do not have preference between link 1 and link 2. The two questions with different travel times were designed to check the consistency in the route choice behavior. They also help to find an average a_1 values for different trip lengths. Survey responses were collected from 202 drivers. There were 404 a_1 values computed from (11). The average values of a_1 is 1.4356. This confirms our assumption that an average driver is risk averse (since a_1>1) in the morning commute to work.

TEST NETWORK

A test network, adopted from *(8)*, is used to illustrate the application the simple ELD function in TransCAD, using the Frank-Wolfe algorithm to solve the DUE problem *(9)*. The network has 25 nodes (of which 4 are O-D nodes), 40 two-way links, t_i^f, one-way link capacity c_i and O-D matrix as given in *(8)*.

The static traffic assignment was first implemented for the DN using the standasrd BPR function, with α=0.15 and β=4. To model driver's route choice in a SN, one only needs to multiply the value of α=0.15 in the BPR function by a_1 to $a_1\alpha$=1.4356x0.15=0.2153.

With the BPR function, there are 10 links with v_i/c_i>1.5. The v_i/c_i of these links have been reduced after the trips are assigned with the ELD function. With the ELD function, risk averse drivers are more sensitive to v_i/c_i (the later is proportional to travel time variation) and therefore they will avoid links which have high volume. There is an overall effect of re-routing some traffic from links with high volumes to links with low volumes, resulting in a more "uniform" distribution of traffic in the network.

The network performance is evaluated by comparing the total vehicle-miles traveled (VMT) and total vehicle-hours traveled (VHT). For the DN, the VMT is 32119 veh-miles and the VHT is 2545.08 veh-hrs. For the SN, the corresponding statistics are 32876 veh-miles and 2425.68 veh-hrs respectively. This reflects the fact that risk averse drivers prefer a longer route with a lower travel time variance than a shorter route with a higher travel time variance. The overall effect of redistribution of flow has resulted in a smaller VHT.

SUMMARY

This paper has derived a simple ELD function, which is of similar form as the BPR function, that represents the route choice behavior of risk averse drivers in a SN. The ELD function has a risk averse coefficient, but it does not depends on the link travel time distribution or variance. This ELD function permits transportation modelers to solve traffic assignment problem in a SN with the familiar DUE algorithms, simply by replacing the BPR function with the ELD function.

A method of calibrating the risk averse coefficient has been proposed and demonstrated with survey data gathered in El Paso, Texas.

The effect of using the ELD function in DUE assignment has been evaluated using a test network. Compare to the results of using the BPR function, the ELD function assigns more trips to low volume route thus results in a more uniform distribution of flow and lower congestion among the links in a network. This leads to lower route travel times for some O-D pairs and an overall reduction in VHT, but at the expense of a higher VMT.

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REFERENCES

1. Chen, A., Z. W. Ji and W. Recker. Travel Time Reliability with Risk-Sensitive Travelers. *Transportation Research Record 1783*, 2002, pp. 27-33.
2. Mirchandani, P. and H. Soroush. Generalized Traffic Equilibrium with Probabilistic Travel Times and Perceptions. *Transportation Science*, Vol. 21, No. 3, 1987, pp. 133-152.
3. Tatineni, M., D. Boyce and P. Mirchandani. Experiments to Compare Deterministic and Stochastic Network Traffic Loading Models. *Transportation Research Record 1607*, 1997, pp. 16-23.
4. Chen, A., M. Tatineni, D-H. Lee, H. Yang. The Effect of the Route Choice Models on Estimating Network Capacity Reliability. *Transportation Research Record 1733*, 2000, pp. 63-70.
5. Tatineni, M. *Solution Properties of Stochastic Route Choice Models*. PhD Thesis, University of Illinois at Chicago, 1996.
6. Cheu, R. L., V. Kreinovich and S. R. Manduva. Traffic Assignment for Risk Averse Drivers in a Stochastic Network. Accepted for Presentation in The 87th Annual Meeting of the Transportation Research Board, January 13-17, Washington D. C.
7. Cheu, R. L., V. Kreinovich, F. Modave, G. Xiang, T. Li, and T. Magoc. How to Estimate, Take Into Account, and Improve Travel Time Reliability in Transportation Networks", Proceedings of the International Workshop on Reliable Engineering Computing REC'08, Savannah, Georgia, February 20-22, 2008 (to appear).
8. Dial, R. B. A Path-Based User-Equilibrium Traffic Assignment Algorithm that Obviates Path Storage and Enumeration, *Transportation Research*, Vol. 40(B), 2006, pp. 917-936.
9. Caliper. *Travel Demand Modeling with TransCAD 4.8*. Caliper Corp., Newton, MA, 2005.