ORIGIN USER EQUILIBRIUM TRAFFIC ASSIGNMENT – A BREAKTHROUGH FOR TRAVEL DEMAND FORECASTING

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1. INTRODUCTION AND OVERVIEW

Recently, a new approach has been proposed for computing static user equilibrium on large on large regional transportation networks. This method, origin user equilibrium (OUE), offers the promise of achieving faster convergence in traffic assignments and also makes it possible to compute to much tighter convergence levels than previously attainable with methods that rely on the Franke-Wolfe (F-W) algorithm or variants thereof. This paper describes OUE and its extension to be useful for planning models, and presents empirical results establishing its properties, uses, and benefits.

Greater convergence is needed for accurately forecasting the impacts associated with road and public transport projects and affects nearly all aspects and components of transportation models as well as being a major determinant of their internal consistency. Congested travel speeds are typically used to compute trip distribution and mode choice, and these speeds will be incorrect if a satisfactory traffic assignment is not achieved. Due to long computational times, many models are insufficiently calibrated and converged for forecasting purposes. This problem is partly the result of and is compounded by the slow convergence of the Frank-Wolfe algorithm.

Origin user equilibrium algorithms have been proposed by Dial (1999, 2006) and Bar Gera (2002). In prior work (Slavin *et al.*, 2006), we implemented an OUE algorithm based on Dial's method and we also tested an executable version of origin-based traffic assignment code written by Bar-Gera. We compared OUE with a very fast traffic assignment which is based upon the Frank-Wolfe algorithm. Concurrent with that prior research, the F-W traffic assignment in TransCAD was multi-threaded leading to speedups in computation proportionate to the number of central processing units and/or CPU cores available. This raised the bar for new assignment algorithms, especially if they are less suitable than F-W for multi-threading. Our testing revealed that OUE can achieve much greater convergence in traffic assignments and that it will be faster to do so, but only when very small gaps are desired.

It also can reach a tight equilibrium even more quickly from a prior solution resulting in much lower computing times for models with feedback and most forecasting tasks. Our testing on a realistic planning model trip table and network suggest that the origin user equilibrium can be deployed by practitioners with immediate benefits in terms of reduced computing times and more tightly converged models.

ORIGIN USER EQUILIBRIUM

In this paper, we focus on the computation of a static user equilibrium (UE) as defined by Wardrop's condition that all used paths for each origin-destination pair have the same minimum cost. In other words, no traveler can switch to a shorter path and improve his or her travel time. In congested networks, user equilibrium is characterized by the use of many paths for many O-D pairs. User equilibrium traffic assignment is the standard method employed in MPO forecasting at present. Bernstein (1990) has shown that UE has good stability with respect to small perturbations; consequently, if a tight equilibrium solution can be computed, it should be a sound method for generating forecasts. Results from Boyce *et al.* (2004) provide empirical support for this conclusion. Achieving much tighter convergence in an equal or lesser amount of computing time would be a breakthrough for practitioners.

The idea behind origin-based assignment is that the equilibrium solution for each origin is an acyclic graph (Jansen and Zozaya-Gorostiza, 1987; Dial, 1999; Bar-Gera, 1999). Origin-based methods maintain acyclic solutions by processing of origin "bushes" or subnetworks (Dial, 1999; 2006). This addresses a major weakness of F-W which has trouble removing cycles once they arise, Origin approaches use this subnetwork, have more efficient shortest path calculations than FW, and prohibit flow from links that are part of cycles giving greater computational efficiency.

To measure convergence, a variety of different measures have been proposed. Rose *et al.* (1985) recommended the relative gap and that is the one we use in this research. The "relative gap" is the difference between the cost of the current UE solution and the cost of the AON solution divided by the cost of the current UE solution. This is a fairly sensitive measure of convergence and is superior to many other stopping criteria such as simple functions of the differences between assignment iterates (Rose *et al.*, 1985).

PROPERTIES OF OUE

To test OUE we had previously used test networks provided by Bar Gera and some other small test cases. Subsequently, we used a large model that we had developed for the Washington, DC metropolitan region. This model was well-calibrated to match counts and is representative of the complexity of models that are in use in many large MPOs. The model has 2500 zones, 6 travel purposes, 3 time periods and feedback is computed through distribution, mode choice, and traffic assignment. Importantly, the model has five assignment classes instead of just one. The traffic assignment was converged to a relative gap of .001 which takes between 80 and 170 iterations of FW for different time periods. Five loops are performed through the model with the result that the skim matrices between the last two loops have a RMSE of less than 1%.

Practitioners are notoriously slow to change their modeling methods, and they will undoubtedly wonder how different the solutions are that generated from the OUE method from those that they currently compute. To address this question, we compared different FW UE solutions for the Washington, D.C. regional network with an OUE solution that was converged to a relative gap of .0000001. As far as we know, this is a much tighter solution than has ever been used in a deployed forecasting model.

The Table below shows the percent root mean square error (RMSE) between the UE and OUE link flows. The first point to note is that highly converged UE and OUE link flows are similar. This is what we would expect if both algorithms converge to the same unique equilibrium point as they should. The UE F-W result at a .00001 gap is reasonably close to the OUE solution at a .0000001 relative gap with an RMSE of just 2.8 percent. The maximum difference in link flows is 172 trips.

COMPARISON OF UE-F-W Link Flows to OUE Link Flows Computed to a 0.000001 Relative Gap– Washington, D.C. Regional Network

GAP	RMSE	Max Flow Difference
0.01	111.214	2719.97
0.001	39.935	2143.85
0.0001	10.15	724.38
0.00001	2.769	171.68

A second important point is that the link flow solution to a UE traffic assignment converged to a relative gap of .01 or 1 percent is quite far away from a highly converged solution. Even at a gap of .001 which probably exceeds the tolerance of nearly all large regional models in the U.S., the differences in link flows can be quite substantial.

Another important feature of our OUE implementation is that it has excellent warm start properties. In other words, a prior solution can be used to compute a new solution even if the network or the trip table have changed somewhat.

When computing feedback in sequential models, the network does not change of course, but there are certainly changes in the trip table in each loop. The ability to save computing time when performing the traffic assignments in each loop is extremely valuable and will encourage planners to achieve greater overall model convergence.

To examine these issues, we computed a traffic assignment on the regional DC network to relative gaps of .001 and .0001 and saved the results. On a 3GHz 4 core Intel Xeon class machine, this took 60 minutes and 215 minutes respectively for FW, but only 12 minutes and 82 minutes using OUE with a warm start. The computational savings illustrated here are dramatic and suggest that OUE will confer significant benefits to practitioners.

CONVERGENCE LEVELS AND PROJECT IMPACTS

Work by David Boyce and his colleagues has stressed the importance of achieving tight convergence in traffic assignments. To investigate his premise and add a further contribution, we studied two changes to the Washington network. The first was an irrelevant change adding a lane to two links in rural Virginia more than 60 miles away from the center of DC. The second was a forecast of the impact of improving the Woodrow Wilson Bridge by adding lanes. We assessed both changes at different levels of convergence. At 1% and .1% relative gaps, the irrelevant change led to differences all over the region. These changes were clearly the result of noise and insufficient convergence. At a gap of .00001, however the only differences are in the immediate vicinity of the links that were changed.

As critical links in the network, the Wilson Bridge is fully utilized when lanes are added. However, the pattern of changes elsewhere is full of noise at .01 and .001 gaps and much more plausible in terms of link utilization at a gap of .00001. Interestingly, calculation of VHT savings from the project suggests that benefit estimation is significantly random at a 1% gap and that tight convergence will give much better estimates of project impacts. Maps illustrating these results are highly instructive and will be presented.

CONCLUSIONS

From our work, we have concluded that origin user equilibrium offers much lower computing times for very small relative gaps. OUE makes it feasible to calculate traffic assignments with gaps of .0001 or lower for virtually all large models in the U.S. There seems to be little risk in deploying OUE because the results will be similar to those obtained by current methods if low relative gaps are achieved.

The warm start properties of OUE are particularly attractive for computing models with feedback and for forecasting project impacts. In the empirical example we considered, using a warm start reduced computing times for traffic assignments to one-fifth of those associated with a regular assignment with a cold start.

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