# Continuous-Time Methods for Modeling Time-of-Day of Travel

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## Background & Objectives

Policy actions such as time-varying pricing schemes aim to achieve congestion relief by re-distributing travel over the day. Realistic assessments of the benefits of such policies require models that can accurately predict the temporal distribution of travel-demand under alternate scenarios. In addition, accurate forecasts of trip-timing decisions (both the time-of-day of travel and the "soak" time, which is the duration for which a vehicle's engine is not operating preceding a successful vehicle start) are also required from the stand-point of air-quality modeling. Thus, development of models for time-of-day of travel continues to be a fertile area of research.

Currently, the common, state-of-practice approach to modeling the time-of-day of travel involves using the discrete-choice methods such as the Multinomial Logit after breaking down the 24-hour day into aggregate periods such as the "morning", "AM peak", mid-day", "PM peak", and "evening". One of the primary issues with this approach is the requirement for apriori discretization of the day into periods. Clearly, what these periods should be need not be unique or readily apparent (especially because individuals do not choose their time of travel from among aggregate periods). Further, such methods might also be restrictive in capturing the temporal shifts in travel patterns in the future. The reader will note that the models capture only shifts *between* the pre-defined time-periods and not shifts *within* time-periods. Finally, each discrete period may involve several hours and this resolution may not be adequate for evaluating dynamic operation strategies (such as time-varying tolls) which may require demand at a finer temporal resolution. Consequently, it would be more appropriate to model time as a continuous entity to overcome issues of apriori discretization and to achieve the finest level of resolution possible to support evaluation of policy actions.

While the conceptual extension of the discrete-choice methodology to accommodate continuous time-of-day choices (or equivalently choices at a very fine resolution of 5 or 15 minutes) is straightforward, practical difficulties have been documented. This is primarily because of the need for a very large number of alternative-specific parameters (constant terms and coefficients on non-time-varying explanatory factors). To address this, functional approximations to alternative-specific parameters have been suggested (Ben-Akiva and Abou-Zeid, 2007; Hess *et al.*, 2005; Vovsha and Bradley, 2004, Cambridge Systematics, 2004; and Guo *et al.*, 2005). As

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an alternative to this approach duration-modeling techniques have also been adopted to model time-of-day on a continuous scale (see for example, Bhat and Steed 2002; Gadda et al., 2007; Komma and Srinivasan, 2008). The objective of this study is to undertake a theoretical and empirical comparison of these two approaches. The next section provides an overview of the two continuous-time modeling methods and describes the comparative evaluation procedure. This is followed by a description of data to be used for the analysis.

## Methodology

This section presents a brief overview of the two approaches for modeling time-of-day choices on a continuous-time scale and subsequently discusses the proposed approach for empirical comparisons. Prior to further discussions, it is useful to note here that both the methods discussed below still involve discretizations of time. However, these discrete periods can be as small as 5 or 15 minutes and hence these models may be construed as continuous-time approaches for all practical purposes. Further, it is also useful to keep in mind that self-reported times from travel surveys are often rounded off to the nearest 5, 10, or 15 minute period and hence it may not be possible to empirically estimate models at any finer resolution.

### MNL with functional approximations to alternative-specific parameters

The utility for discrete period *t* and decision maker *q* is given by:

$$U_{qt} = V_{qt} + \varepsilon_{qt} \quad \forall t \in T$$

$$V_{qt} = \alpha_t + \beta_t X_q + \gamma Z_{qt}$$

Where

$X_q =$ Non - time varying explanatory variables

$Z_{qt} =$ Time varying explanatory variables (example travel time, cost) at discrete period $t$

$\alpha_t =$ Alternative specific constant for discrete period $t$

$\beta_t =$ Coefficients on non time varying explanatory variables for discrete period $t$

$\gamma =$ Coefficients on time - varying explanatory variables

As the number of discrete time periods (*t*) can be numerous considering that each discrete period is of very short duration, the number of alternative-specific parameters $(\alpha_t, \beta_t)$ to be estimated can be very large. To address this issue, functional approximations may be adopted. One such approximation (in the case of the constant terms) could be:

$$\alpha_t = \begin{cases} \delta_1 \sin\left(\frac{2\pi t}{24}\right) + \delta_2 \sin\left(\frac{4\pi t}{24}\right) + \delta_3 \sin\left(\frac{6\pi t}{24}\right) + \delta_4 \sin\left(\frac{8\pi t}{24}\right) + \\ \lambda_1 \sin\left(\frac{2\pi t}{24}\right) + \lambda_2 \sin\left(\frac{4\pi t}{24}\right) + \lambda_3 \sin\left(\frac{6\pi t}{24}\right) + \lambda_4 \sin\left(\frac{8\pi t}{24}\right) \end{cases}$$

If the choice set comprises 96 15-minute intervals (totaling 24 hours), then the above approximation reduces the number of alternative-specific constants from 95 to 8.

The probability that decision maker $q$ chooses discrete period $t$ is then be given by the logit-formula:

$$\Pr ob_q(t) = \frac{\exp(V_{qt})}{\sum_{k \in T} \exp(V_{qk})}.$$

<u>Hazard-duration model</u>

The "hazard" for departing at any time of the day $u$ (measured on a continuous scale, say in minutes from 3 AM) is defined as the probability that a person will depart immediately after time $u$ conditional on not departing until time $u$. This hazard is assumed to have the following functional form:

$$\lambda(u) = \lambda_0(u) \exp\left(\beta X + \gamma Z\left(u\right)\right) w$$

In the above equation, $\lambda_0(u)$ is the baseline hazard. $X$ and $Z(u)$ are vectors of non-time varying and time varying covariates respectively. For example, $X$ could include the socio-demographic characteristics of the worker whereas $Z(u)$ includes the travel times between home and work locations at time $u$. $\beta$ and $\gamma$ are the vector of coefficients on the non-time varying and time varying covariates respectively. $w$ is the unobserved heterogeneity term assumed to follow a gamma distribution (with variance = $\sigma^2$) and independent of the covariates.

We adopt a non-parametric distribution for the baseline hazard (i.e., $\lambda_0(u)$) in our specification. For this purpose, we discretize the continuous time into $K$ unique time intervals. Let $p$ denote the index for the time intervals ($p$ = 1, 2,…, $K$) and $a_p$ represent the upper bound time corresponding to discrete interval $p$. Therefore discrete period $p$ represents the time interval [ $a_{p-1}$, $a_p$ ] and the duration of this discrete period is given by, $\Delta_p = a_p - a_{p-1}$. The baseline hazard is then assumed to be a constant within each of these discrete periods (*i.e.*, $\lambda_0(u)$ = exp( $\delta_p$ ) if $u$ element of discrete period $p$). In addition, we assume that the value of time-varying covariates remain constant within each discrete time period (*i.e., Z(u)* = $Z_p$ if $u$ element of discrete period $p$).

The unconditional probability of departure in interval $p$ is given by (See, Bhat and Steed, 2002 for details)

$$\Pr ob[t = p] = \left[1 + \sigma^2 \left\{ \sum_{j=0}^{p-1} \Delta_j \exp\left(\delta_j + \beta X + \gamma Z_j\right) \right\} \right]^{-\sigma^{-2}} - \left[1 + \sigma^2 \left\{ \sum_{j=0}^{p} \Delta_j \exp\left(\delta_j + \beta X + \gamma Z_j\right) \right\} \right]^{-\sigma^{-2}}$$

Where $\delta_0 = -\infty$ and $\delta_K = +\infty$.

Based on the brief descriptions from above, it may be noted that the first model is based on the theory of utility maximization whereas the second is not. Further, the first model assumes that the probability of departure at any time period is a function of the travel times prevailing during all times of the day whereas in the second model, this probability is only a function of travel times prevailing at all times *until* the discrete time interval under consideration and does not depend on the travel times *after* the time interval under consideration (although this can be addressed by introducing future travel times as explanatory variables, see for example, Komma and Srinivasan, 2008). Finally, unlike the hazard structure, the first model does not recognize the inherent ordering of the choice alternatives and its MNL structure leads to the IIA property.

In this research, these two approaches will be empirically compared in the context of time-of-day choices for home-to-work commute travel. Specifically, empirical models using the two approaches will be estimated using the same dataset and compared. Further, the models will also be compared in terms of their ability to predict the time-of-day choices on a validation sample. Finally, aggregate shifts in the temporal demand patterns as a consequence of travel time changes will also be examined using hypothetical simulations.

**Data**

The San Francisco Bay Area Travel Survey (BATS) conducted in the year 2000 by is the primary source of data used in this study. In this survey, detailed activity-travel and socio-economic information was collected from 33402 members (14529 households) for a two-day period. These data were augmented with land-use and inter-zonal level-of-service information obtained from Bay Area Metropolitan Transportation Commission (MTC). The characteristics of the Traffic Analysis Zones (TAZs) like the population and employment densities, area type (CBD, Urban, Suburban, and Rural), and land-use mix indicators were included in the land-use file. The inter-zonal level-of-service file provides data on the network characteristics such as distance, travel time, and costs for the peak and off-peak periods. The final estimation dataset comprises of 4661 commute journeys to work obtained from 3162 persons (fully flexible, full-time workers) and 2894 households.

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