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# Discrete Network Design Problems Accounting for Dynamic Traffic Assignment Conditions

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**Abstract**

This paper introduces a new heuristic approach for improving the computational efficacy of a discrete network design model that accounts for User Equilibrium Dynamic Traffic Assignment (UE-DTA) conditions, by employing genetic algorithm (GA). The cell transmission (CTM) based DTA model is used to reflect traffic realities and capture traffic dynamics. The mathematical model of this problem consists of two level problems: upper-level and lower-level problems. The upper-level problem is to create feasible network design combinations under a budget constraint using GA and the lower-level problem is for the network evaluation using CTM-based DTA. Extensive experimental design and procedure on GA operators and parameters are conducted to find the appropriate operator and parameter. Statistical tests are performed for model validation. The computational complexity analysis has shown that the proposed approach could significantly reduce the computational burden without losing solution quality.

## 1. INTRODUCTION AND BACKGROUND

The main purpose of the transportation network modelling is to design transportation networks to achieve a desirable level of network performance. Most of traditional Network Design Problems (NDPs) and its solution algorithms (Magnanti and Wong, 1984; Bell and Iida, 1997) were relating to the Continuous NDP (CNDP) dealing with road expansions due to problem complexity of Discrete NDP (DNDP) such as an addition of lane, a road closure scheme, and the provision of a new public transport service. The analytical optimization methods to solve DNDP works well for only a very small scale network, because DNDP is non-convex and combinatorial problems that usually lead to NP-hard problem. To overcome these limitations of existing solution algorithms to DNDP, many researchers have proposed the innovative solution search techniques such as Genetic Algorithm (GA) (Holland, 1975; Goldberg, 1999), tabu search (Glover, 1989), and simulated annealing (Kirkpatrick, 1984).

The heuristic approaches to solve DNDP using GA and tabu search were proposed by Mouskos (1991) and Xiong and Schneider (1992), respectively. In their experiments, they evaluated the networks under Deterministic User Equilibrium (DUE) conditions that did not account for traffic realities in the traffic assignment procedure, and used Volume-to-Capacity (V/C) ratio when selecting candidate links. Recently, Jeon *et al.* (2005, 2006) and Jeon (2005) proposed a selectorecombinative GA-based approach for relaxing computational complexity of DNDP under Deterministic User Equilibrium (DUE) conditions. Unlike existing deterministic DNDP, this model proposed uses the density-to-jam density (D/J) ratio, which is analogy to V/C, but can exactly capture the traffic conditions, by accounting for traffic realities such as link spillover and shockwave propagation through the Dynamic Traffic Assignment (DTA).

DNDP can be formulated as only a Bi-Level Programming Problem (BLPP) or a Stackelberg game (von Stackelberg, 1934). A Stackelberg game is known as a leader-follower game. The interaction between network design authority (i.e., the leader) and network users (i.e., the follower) is a sort of game between two players: the leader and the follower. Even though the follower consists of collective users, it can be treated as one player (Chen, 1998). The network user is trying to minimize his/her own travel time by choosing his/her best route, while the network design authority is trying to maximize a transportation system performance in terms of Total System Travel Time (TSTT). In case of a Stackelberg game (e.g., UE-DNDP), the leader controls the design variable to maximize transportation system performance in terms of users' total travel times resulting in the leader's strategy (i.e., network design combination). When evaluating user behaviors in networks, accounting traffic realism in network evaluation is a critical issue. CTM-based UE-DTA simulation model developed by Ziliaskopoulos and Lee (1996) is used to solve the lower-level problem. Candidate links to be improved are selected based on D/J parameter, which is generated from the CTM-based DTA model.

Furthermore, since the well-known Braess' paradox (Braess, 1968) is often observed in DNDP, it needs to search exhaustive feasible design combinations. To search an exhaustive solution space efficiently, this study employed GA as a heuristic solution search method, which can search and manage multiple solutions simultaneously. The purposes of this study are (i) to develop a GA-based heuristic solution search methodology for discrete transportation network design problems accounting for DTA conditions, (ii) suggest a new measure of effectiveness such as D/J used in choosing candidate links, and (iii) reduce the computation time when evaluating DNDP.

## 2. MATHEMATICAL FORMULATIONS OF DISCRETE NETWORK DESIGN PROBLEMS

Mathematical formulations for BLPP of UE-DNDP. BLPP arises when two independent decision makers ordered within a hierarchical structure have their own objectives (Gümüş and Floudas, 2001). The decision maker (*i.e.,* users or followers) at the lower-level tries to achieve his/her objective under the given parameters (*i.e.,* network design combinations) from the upper-level decision maker *(i.e.,* leader or network design authority) accounting for budget limit and with complete information on the reactions of network users (Hansen *et al.,* 1992). In this context, BLPP for UE-DNDP is considered as a Stackelberg game.

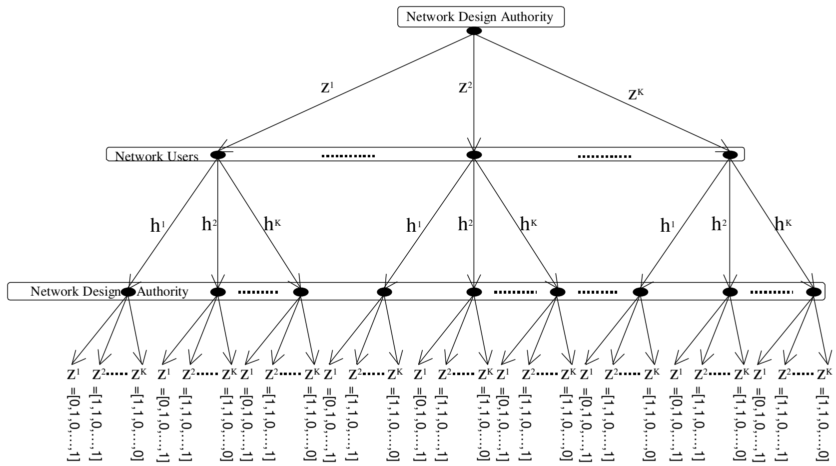


**Figure 1 Stackelberg game between network design authority and network users**

The network design authority at the first stage of the game initiates the move by setting the design solution, $\mathbf{z}$. When users make their move at the second stage of the game, they will choose the optimal flow $\mathbf{h}$ to solve the UE-DTA problem. The equilibrium of such a Stackelberg game shown in Figure 1 can be defined as follows:

*Stackelberg Equilibrium: In the non-cooperative game between the network design authority and the network users, the strategy combination (* $\mathbf{z}$ *,* $\mathbf{h}$ *) is a Stackelberg equilibrium if and only if it solves the following bi-level programming problem (BLPP):*

$$U:\left\{\begin{array}{lll} \underset{\mathbf{z}^{\kappa}\in\mathbf{Z}}{Minimize} & T^{\kappa}(\mathbf{z}^{\kappa},\mathbf{h}^{\kappa}(\mathbf{z}^{\kappa})) & \\ Subject\ to: & G^{\kappa}(\mathbf{z}^{\kappa},\mathbf{h}^{\kappa}(\mathbf{z}^{\kappa}))\leq 0 & \forall\kappa\in \mathrm{K} \end{array}\right\} \tag{1}$$

where $\mathbf{h}^{\kappa}=(\mathbf{x}^{\kappa},\mathbf{y}^{\kappa})$ is optimal for

$$L:\left\{\begin{array}{ll} \underset{\mathbf{x}^{\kappa}\in\mathbf{X},\mathbf{y}^{\kappa}\in\mathbf{Y}|\mathbf{z}^{\kappa}\in\mathbf{Z}}{Minimize} & f(\mathbf{z}^{\kappa},\mathbf{x}^{\kappa}(\mathbf{z}^{\kappa}),\mathbf{y}^{\kappa}(\mathbf{z}^{\kappa})) \\ Subject\ to: & g(\mathbf{z}^{\kappa},\mathbf{x}^{\kappa}(\mathbf{z}^{\kappa}),\mathbf{y}^{\kappa}(\mathbf{z}^{\kappa}))\leq 0 \\ & \mathbf{z}^{\kappa}\in\mathbf{Z},\mathbf{x}^{\kappa}\in\mathbf{X},\mathbf{y}^{\kappa}\in\mathbf{Y} \end{array}\right\} \tag{2}$$

Here, $T^{\kappa}(\cdot)$ refers to TSTT of total network users under $\kappa^{th}$ network design combination. The vector $\mathbf{z}^{\kappa}$ consisting of binary numbers (i.e., 0s and 1s) refers to $\kappa^{th}$ network design combination. $\mathbf{h}^{\kappa}=(\mathbf{x}^{\kappa},\mathbf{y}^{\kappa})$ refers to the best user behavior according to the network design combination. The objective of upper-level problem is to select the optimal network design combination satisfying with not only budget constraint but also minimum TSTT.

Since the upper-level (*U*) is a discrete problem that means it is not differentiable, and BLPP is difficult to solve in analytical methods. This means that universal solution algorithms do not exist accordingly even though the lower-level (*L*) is a differentiable problem. The major difficulty in solving BLPP is that the response function of the user to the decisions of the design authority might not be defined uniquely. Furthermore, the overall formulation of these interactions between the design authority and users is a non-convex and non-smooth model (Breiner and Avriel, 1999). Therefore, it needs to use an efficient search algorithm like SGA to search the network design combination in the upper-level problem, while CTM-based DTA model is used to evaluate the lower-level problem.

## 3. SOLUTION PROCEDURE

***Step 0*: *Initialization***: Input O-D demand and network, and *Gen* =0, *pop*=0

***Step 1*: *Generate the initial binary chromosome and population***

- ***Step 1.1***: Perform a simulation-based DTA model with initial conditions (no network improvement)
- ***Step 1.2***: Generate the cumulative cell density and flow for aggregation time intervals during peak period
- ***Step 1.3***: Select cells in which maximum cell density-to-jam density ratio within the link is greater than specified criteria
- ***Step 1.4***: Map the cells selected in ***Step 1.3*** onto the concerned links
- ***Step 1.5***: Select *L* links which are most congested links and chromosome length
- ***Step 1.6***: Generate the initial population
  - ***Step 1.6.1***: *pop = pop* + 1
  - ***Step 1.6.2***: Generate the binary chromosome randomly
  - ***Step 1.6.3*****:** Check a budget limit
  - ***Step 1.6.4*****:** If a chromosome satisfies with a budget limit and *pop* < *Popsize*, go to ***Step 1.6.1***; otherwise repeat ***Step 1.6.5*** until satisfying a budget limit
  - ***Step 1.6.5*****:** Change a gene into "0" by random selection of gene, go to ***Step 1.6.3***

***Step 2*: *Genetic algorithm procedure***

- ***Step 2.1*****:** *Gen=pop=0*
- ***Step 2.2*****:** *Gen=Gen* + 1
- ***Step 2.3*****:** Create population of next generation
  - ***Step 2.3.1*****:** pop=pop + 1
  - ***Step 2.3.2*****:** Select the two parents from the previous generation according to selection methods
  - ***Step 2.3.3*****:** Do crossover operations
    - ***Step 2.3.3.1*****:** Check a budget limit
    - ***Step 2.3.3.2*****:** If a chromosome satisfies with a budget limit, go to ***Step 2.3.4***; otherwise repeat ***Step 2.3.3.3*** until satisfying a budget limit
    - ***Step 2.3.3.3*****:** Change a gene into "0" by random selection of gene, go to ***Step 2.3.3.1***
  - ***Step 2.3.4*****:** Compare the fitness values of offspring and parent
  - ***Step 2.3.5*****:** Keep the better one
  - ***Step 2.3.6*****:** If *pop* < *Popsize*, go to ***Step 2.3.1***; otherwise go to ***Step 2.4***
- ***Step 2.4*****:** Calculate the fitness values $f_2(Gen,Popsize)$ for the population of generation *Gen*
- ***Step 2.5*****:** Select best *K* chromosomes and store them onto the best chromosome database to be evaluated by a simulation-based DTA model
- ***Step 2.6*****:** Check convergence criterion (*i.e.,* $f_2(Gen,1)=\ldots=f_2(Gen,Popsize)$). If convergence criterion satisfies, then go to ***Step 3***; otherwise go to ***Step 2.2***

***Step 3*: *Evaluate the feasible chromosome using a simulation-based DTA model***

- ***Step 3.1***: κ=1, *Gen*=0
- ***Step 3.2***: Select best *K* chromosomes at generation *Gen*
- ***Step 3.3***: Evaluate the updated network based on best *K* chromosomes
  - ***Step 3.3.1***: Conduct a CTM-based DTA model with the $\kappa^{th}$ improved network
  - ***Step 3.3.2***: If κ ==*K*, then go to ***Step 3.4***; otherwise κ=κ+1 and go to ***Step 3.3.1***
- ***Step 3.4***: Select the best chromosome with $f_1(Gen)$=minimum TSTT among the best *K* chromosomes, regardless $f_2(Gen)$
- ***Step 3.5***: If *Gen==Maximum generation*, then go to ***Step 3.7***; otherwise go to ***Step 3.6***
- ***Step 3.6***: *Gen*=*Gen*+1, then go to ***Step 3.2***
- ***Step 3.7***: Select the best network design combination among the best chromosomes for all the generations
- ***Step 3.8***: Stop

## 4. GA PARAMETERS AND OPERATORS

The overall procedure for experimental design and numerical tests of GA operators and parameters and to choose GA parameters and operators follows the procedure of Jeon *et al* (2006). For this test, a aggregate Sioux Falls networks (76 arcs, 24 nodes and 552 origin-destinations pairs) was used. A simulation time is a 6600 [2200:2200:2200] seconds with a 6 seconds time interval. It is assumed that travel demand is distributed in the ratio of 0.2:0.6:0.2 over [2200:2200:2200] seconds time period, respectively, to replicate peak hour conditions to initial candidate links selected from the initial UE-DTA without network improvement conditions.

To select appropriate GA operators, the preliminary test was conducted. Each gene in chromosome corresponds to each link. The gene consists of binary strings 0s or 1s signifying that the link capacity is improved if gene is '1,' and '0' otherwise. A chromosome length ($L$) could be up to 76 genes (i.e., number of maximum links), but for the sake of convenience 30 genes are considered through the initial dynamic traffic assignment without network improvement. In addition, it is assumed that a given budget is \$20 Million, and the cost for adding unit increase of link capacity is approximately \$2 Million per mile.

Extensive experimental design and test have been conducted with a crossover probability 0.95, tournament sizes (s=2~14), and crossover types (two-point and uniform). Through this test, a tournament selection without replacement option showed the best fitness value with uniform crossovers, without mutation, and s>10. As shown in Figure 2, the optimal population size (Popsize=250) was found through network evaluation with different population sizes (Popsize=100~300), Pc=0.95, s=14, and uniform crossover. Figure 2 also showed that selection rate (SR) 20% of chromosomes could be used for network evaluation without losing solution quality in terms of Total System Travel Time (TSTT).

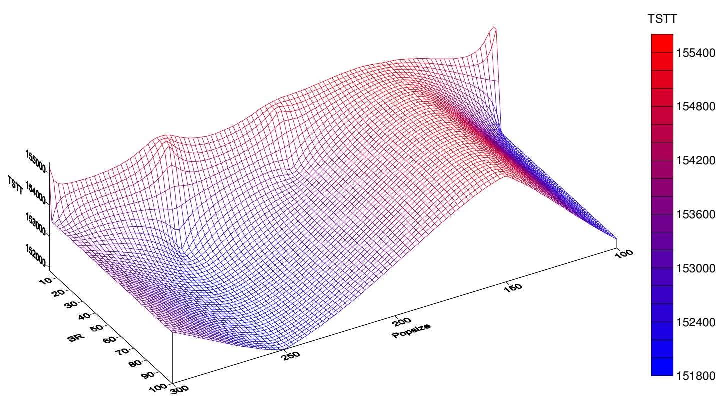


**Figure 2 Contour plot of TSTT vs Selection ratio and Population size**

The effect of a population size factor is statistically significant since $F$ =24.610>2.447= $F_{0.05}(4,120)$. In this test, the case of *Popsize*=250 provides a better solution in terms of TSTT on average than other population sizes.

The effect of interaction between selection rate and population size factors is not statistically significant since $F$ =0.091<1.478= $F_{0.05}(44,120)$. Therefore, there is no interaction effect between selection rate and population size factors in TSTT.

## 6. MODEL VALIDATION

For model validation, this study uses the Exhaustive Search Method (ESM) for UE-DNDP with a small network to examine and validate the quality of the network design solution of SGA-based UE-

DNDP model. For validation, five different test networks with two different demand levels for each network shown in Table 1. In this test, *SR*=20% is used for evaluating improved networks.

**Table 1 Small test networks for model validation**

| Test network # | # of nodes | # of links | # of origins | # of destinations | Total demand level | Budget level ($,Million) |
|---|---|---|---|---|---|---|
| Test-net1-1 | 12 | 32 | 12 | 12 | 4252 | 2.0 |
| Test-net1-2 | 12 | 32 | 12 | 12 | 8304 | 2.0 |
| Test-net2-1 | 8 | 20 | 8 | 8 | 2734 | 2.0 |
| Test-net2-2 | 8 | 20 | 8 | 8 | 5468 | 2.0 |
| Test-net3-1 | 10 | 26 | 10 | 10 | 4514 | 2.0 |
| Test-net3-2 | 10 | 26 | 10 | 10 | 9028 | 2.0 |
| Test-net4-1 | 12 | 34 | 12 | 12 | 5828 | 2.0 |
| Test-net4-2 | 12 | 34 | 12 | 12 | 11656 | 2.0 |
| Test-net5-1 | 8 | 22 | 8 | 8 | 2188 | 3.0 |
| Test-net5-2 | 8 | 22 | 8 | 8 | 4376 | 3.0 |

Since all the calculated t-stat values are less than $t_{0.025}(9)$=2.262, the null hypothesis is not rejected. From this test, with 9 degrees of freedom, we do not reject $H_0$: $\overline{\text{TSTT}}_{\text{SGA-based}} = \overline{\text{TSTT}}_{\text{ESM-based}}$ at the α=0.05 significance level. In other words, we can assume the average minimum TSTT resulting in SGA equals to the average minimum TSTT resulting in ESM under the same network evaluation environment. The solution accuracy in terms of TSTT was as high as 99 % (*i.e.,* average relative difference of 1 %) from the model validation with small network. Since the CTM-based DTA model used here adopts a mesoscopic model based on CTM to propagate traffic flow on networks, the model makes it possible to capture more precise traffic dynamics.

## 7. SUMMARY AND CONCLUSIONS

A bi-level UE-DNDP model accounting for dynamic traffic assignment conditions was formulated and proposed the SGA-based solution search procedure for solving UE-DNDP model. The SGA-based solution search procedure was developed to create feasible design combinations by accounting for a budget limit. The interface to integrate with SGA model and a mesoscopic simulation model was developed to simulate the traffics based on the networks generated by SGA model and capture interactions between vehicles, accounts for link spillovers and shockwaves while assigning traffic. The density-to-jam density ratio (D/J) was proposed as a new measure of effectiveness, rather than V/C used in the deterministic user equilibrium model. Accordingly, the proposed model overcame the problem that one flow state corresponding to two different traffic conditions (congested and uncongested states) which were not accounted in the deterministic user equilibrium model.

We found that the proposed model could find a reasonable solution by evaluating DNDP with best 20% chromosomes of population. The solution accuracy in terms of TSTT was as high as 99% from the model validation. Furthermore, the statistical test showed that there was no significant difference between $\overline{\text{TSTT}}_{\text{ESM-based}}$ and $\overline{\text{TSTT}}_{\text{SGA-based}}$. Therefore, the proposed solution procedure could provide a reliable solution for the complex DNDP. Furthermore, the proposed search procedure can improve the search time up to 5 times due to SR=20%.

**REFERENCES**

1. Bell, M.G.H. and Iida, Y., 1997. *Transportation Network Analysis*. John Wiley & Sons Inc., 605 Third Avenue, New York, NY 10158-0012, U.S.A.
2. Braess, D., 1968. Uber ein paradoxen der verkehrsplanung, *Unternehmensforschu*, Vol. 12, pp. 258-268
3. Breiner, A. and Avriel, M., 1999. Two-Stage Approach for Quantitative Policy Analysis Using Bilevel Programming. *Journal of Optimization Theory and Applications*, Vol. 100, No. 1, pp. 15-27
4. Chen, O.J., 1998. *Integration of Dynamic Traffic Control and Assignment*, Ph.D. Dissertation, MIT
5. Daganzo, C.F., 1993. *The Cell Transmission Model Part I: A Simple Dynamic Representation of Highway Traffic.* UCB-ITS-PRR-93-7, ISSN: 10551425, California PATH Program, Institute of Transportation Studies, University of California, Berkeley, CA
6. Daganzo, C.F., 1994. *The Cell Transmission Model: Network Traffic.* California PATH Working Paper UCB-ITS-PWP-94-12, Department of Civil Engineering and Institute of Transportation Studies, University of California, Berkeley, CA
7. Deb, K., 2001. *Multi-Objective Optimization using Evolutionary Algorithms*. John Wiley & Sons, Ltd., ISBN: 0-471-87339-X
8. Gerlough, D.L. and Huber, M.J., 1975. *Traffic Flow Theory: A Monograph, TRB Special Report 165*, Transportation Research Board, National Research Council, Washington, D.C., 1975
9. Glover, F., 1989. Tabu Search: Part I. *Journal on Computing, Operations Research Society of America*, Vol. 1, No. 3, pp. 190-206
10. Goldberg, D.E., 1989. *Genetic Algorithm in Search, Optimization and Machine Learning*. Addison-Weslely
11. Goldberg, D.E., 1999. The Race, the Hurdle, and the Sweet Spot: Lessons from Genetic Algorithms for the Automation of Design Innovation and Creativity. *Evolutionary Design by Computers*, edited by Peter J. Bentley, pp. 105-118, Morgan Kaufmann Publishers, Inc.
12. Gümüş, Z.H. and Floudas, C.A., 2001. Global Optimization of Nonlinear Bilevel Programming Problems. *Journal of Global Optimization*, Vol. 20, pp. 1-31
13. Hansen, P., Jaumard, B. and Savard, G., 1992. New Branch-and-Bound Rules for Linear Bilevel Programming, *SIAM Journal on Scientific and Statistical Computing* 13, pp. 1194–1217
14. Holland, J.H., 1975. *Adaptation in Natural and Artificial System*. University of Michigan Press, Ann Arbor, MI
15. Jeon, K., Ukkusuri, S. and Waller, S.T., 2005. Heuristic Approach for Discrete Network Design Problem Accounting For Dynamic Traffic Assignment Conditions: Formulations, Solution Methodologies, Implementations and Computational Experiences. Presented at *84th Annual Meeting of the Transportation Research Board*, Washington, D.C.
16. Jeon, K., Lee, J., Ukkusuri, S. and Waller, S.T., 2006. New approach for relaxing computational complexity of discrete network design problem using selectorecombinative genetic algorithm. Accepted for publication in *Transportation Research Record*, Washington D.C.
17. Jeon, K., 2006. *Genetic Algorithm Based Search Strategy for Discrete Network Design Problems Accounting for Dynamic Traffic Assignment Conditions: Formulation, Search Strategy, Implementation, and Validation*, Ph.D. Dissertation, The University of Illinois at Urbana-Champaign, Urbana, IL
18. Kirkpatrick, S., 1984. Optimization by Simulated Annealing: Quantitative Studies. *Journal of Statistical Physics*, Vol. 34, pp. 975-986
19. Magnanti, T.L. and Wong, R.T., 1984. Network Design and Transportation Planning Models and Algorithms. *Transportation Science*, Vol. 18, No. 1, pp. 1-55
20. Mouskos, K.C., 1991. A Tabu-Based Heuristic Search Strategy to Solve a Discrete Transportation Equilibrium Network Design Problem, *Ph.D. Thesis*, The University of Texas at Austin, TX
21. Papacostas, C.S. and Prevedouros, P.D., 1993. *Transportation Engineering and Planning*, Prentice Hall, Second Edition.
22. Papadimitriou, C.H. and Steiglitz, K., 1998. *Combinatorial Optimization: Algorithms and Complexity.* Dover Publications, Inc., Mineola, NY

23. Ukkusuri, S., 2002. *Linear Programming for the User Optimal Dynamic Traffic Assignment Problem*, Master Thesis, University of Illinois at Urbana-Champaign, IL
24. Ukkusuri, S. and Waller, S.T., 2004. Linear Programming Models for the User and System Optimal Dynamic Network Design Problem: Formulations, Implementations and Computations. *Transportation Research Board 2004 Annual Meeting CD-ROM*
25. Waller, S.T., 2000. Optimization and Control of Stochastic Dynamic Transportation Systems: Formulations, Solution Methodologies, and Computational Experience, *Ph.D. Thesis in Industrial Engineering and Management Sciences*, Northwestern University, Evanston, IL
26. Xiong, Y. and Schneider, J.B., 1995. Processing of Constraints in Transportation Network Design Problem. *Journal of Computing in Civil Engineering*, Vol. 9, No. 1, pp. 21-28
27. Yang, H. and Bell, M.G.H., 1998. Models and Algorithms for Road Network Design: A Review and Some New Development. *Transport Review*, Vol. 18, No. 3, pp. 257-278
28. Ziliaskopoulos, A.K. and Lee, S., 1996. Cell Transmission Based Assignment-Simulation Model for Integrated Freeway/Surface Street Systems. *Presented in Transportation Research Board Annual Conference*, Washington D.C.