Title: Integrating Transit Travel Behavior and Urban Form

Introduction

This study presents a behavioral model of transit patronage in which residential location, travel behavior, the spatial dispersion of non-work activities, and urban form are all endogenously determined. In a departure from the monocentric urban monocentric model, residential location is defined as the optimal *job-residence pair* in an urban area in which jobs and residences are dispersed. Following urban residential location theory, the location decision is assumed to be the result of a trade-off between housing expenditures and transportation costs, given income and the mode-choice set. The location decision is also based on idiosyncratic preferences for location and travel. In addition to determining optimal residential location, this approach also determines the optimal sequence of non-work trip chains, goods consumption, and transit patronage.

In this model, travel demand is considered an *indirect* demand brought about by the necessity to engage in out-of-home activities whose geographical extent is affected by urban form. Furthermore, budget-constrained utility maximizing behavior leads to an optimization of the spatiotemporal allocation of these activities and an optimal number of chained trips. Socio-demographic factors directly influence residential location, consumption, and travel behavior.

Within this framework, the study addresses questions related to the interrelation between urban form, residential location, and transit demand. How do location decisions affect travel behavior? How does urban form relate to travel behavior? Do residential location and urban form affect travel behavior? What is the impact of higher density on travel behavior? To address these questions, this research begins with a travel-demand model treating residential location and density as exogenous. Subsequent extensions that relax these assumptions are then introduced and expected behavioral conclusions are reached. In its final specification, the model treats as endogenous transit station proximity to the residence. Transit supply is introduced through station proximity. Incorporation of transit supply and demand produces a general equilibrium model of transit.

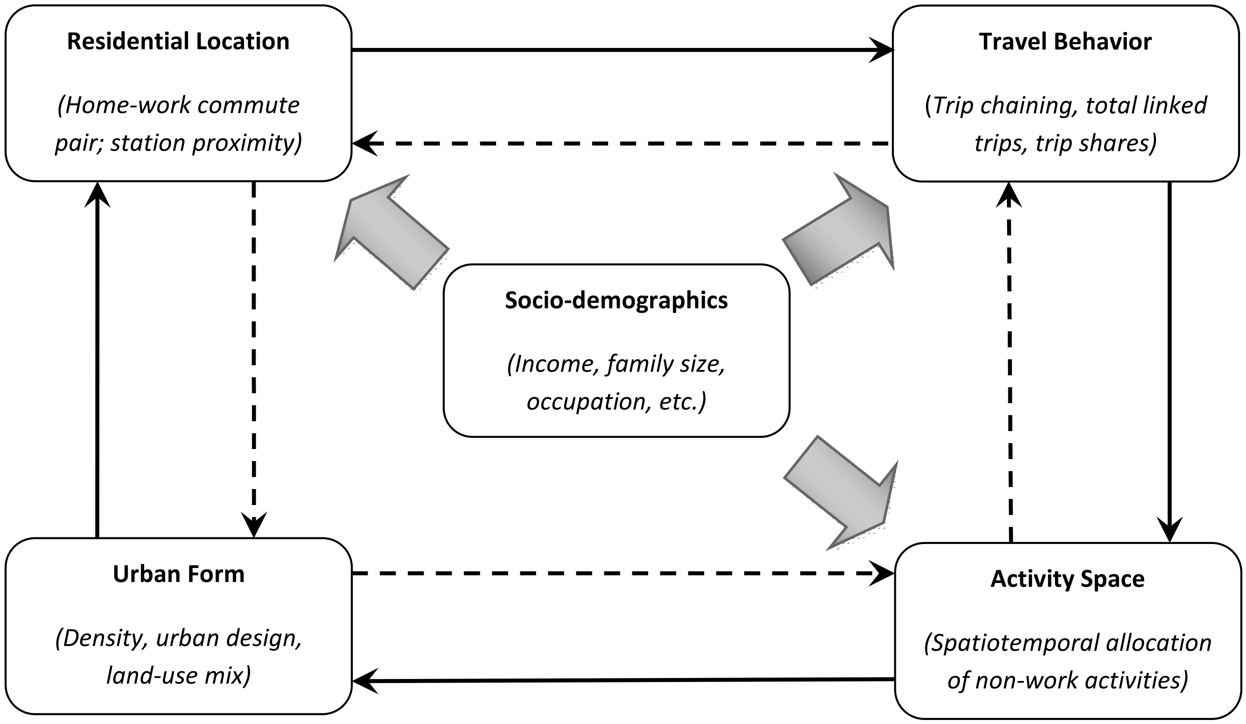
The research reported here is the first example of empirical work that explicitly relates location to trip chaining behavior in a context where individuals jointly decide location, the optimal trip chain, and the area of non-work activities, based on the optimal trade-off between commute time, leisure, and non-work travel activities.

Conceptual Model

We consider the demand for travel a derived demand brought about by the necessity to engage in out-of-home activities whose geographical extent is affected by urban form. Furthermore, budget-constrained utility-maximizing behavior leads to an optimization of the spatiotemporal allocation of these activities and an optimal number of chained trips. Socio-demographic factors directly influence residential location, consumption, and travel behavior. To date, no empirical work has been done that explicitly relates location to trip chaining behavior in a context where individuals jointly decide location, the optimal trip chain, and the area of non-work activities, based on the optimal trade-off between commute time, leisure, and non-work travel activities.

Figure 3.1 displays a conceptual representation of the relationship between urban form, residential location, and the demand for travel. While this framework is suited to explain the determinants of the demand for travel in general, the focus here is on the demand for transit services.

FIGURE 3.1 Conceptual Model of Urban Form and Travel Behavior



In this model, residential location, travel behavior, the activity space, and urban form are all endogenously determined. Following urban residential location theory, the location decision is assumed to be the result of a trade-off between housing expenditures and transportation costs, given income and the mode-choice set. In a departure from the monocentric model, the definition of residential location is taken from the polycentric model of Anas and his associates (Anas and Kim 1996; Anas and Xu 1999). The model is generalized and presented in Appendix C. Residential location is defined as the optimal *job-residence pair* in an urban area in which jobs and residences are dispersed. Following Anas, the location decision is also based on idiosyncratic preferences for location and travel. In addition to determining optimal residential location, this approach also determines the optimal sequence of non-work trip chains, goods consumption, and transit patronage.

Model I: Exogenous Residential Location and Population Density

In this specification, residential location, transit station proximity, and density are exogenous. Given these variables, the model jointly determines the activity space and the optimal trip chain. The optimization (not explicitly modeled here) determines a travel demand function, given consumption and location decisions. The household (rather than the individual) is the unit of analysis because these decisions take place at the household level. Empirical studies on the relevance of transit station proximity to transit patronage show a strong relationship between transit use and station proximity (Cervero 2007; Cervero and Wu 1998). Therefore, this model includes this possibility. To include these considerations, Model I takes the following specific form

$$TC = TC(AS, RL, WD, X_{TC}) \tag{3.1}$$

$$AS = AS(TC, D, X_{AS}) \tag{3.2}$$

$$TD = TD(TC, AS, RL, WD, X_{TD}) \tag{3.3}$$

where

TC = the number of non-work trips per commute-chain

AS = the activity space (measured as the geographic area surrounding the residence in which non-work trips are made)

TD = the demand for transit trips (measured as the number of transit trips)

RL = residential location (measured as the *job-residence pair* distance)

D = a vector of residential and employment density controls

WD= transit station proximity (measured as walking distance to the nearest transit station)

X_{TC} = a vector of controls specific to the TC function;

X_{AS} = a vector controls specific to the AS function

X_{TD} = a vector of controls specific to the TD function

This model permits testing the hypothesis that individuals living farther from the workplace engage in more complex tours characterized by a higher number of non-work trips linked to the commute tour. In addition, trip chaining, as it relates to transit patronage, is directly affected by transit station proximity and by other factors summarized by the vector of controls, X_{TC}. This vector, as explained in more detail in Chapter 4, includes vehicle availability and the presence of young children among other factors likely to affect trip-chaining formation.

Trip-chaining behavior defines an activity space, AS, which is assumed to represent the optimized spatiotemporal allocation of non-work activities as affected by the built environment, summarized by the exogenous vector, D. For example, more densely populated urban areas have more densely clustered activity locations, which shrink the size of the activity space relative to less densely populated areas. A smaller activity space reduces trip chaining, TC, ultimately affecting the demand for travel, TD.

The model may be used to test the effect of urban design policies directly affecting travel distances and the land-use mix. In general, it may be used to test if higher density environments entail shorter travel distances, which in turn should affect the composition and complexity of trip chains and the overall amount of travel.

Residential Location, *RL*, and Transit Station Proximity, *WD*

The definitions of residential location and transit station proximity used here differ from those used in the current literature. For example, in studies of residential self-selection, the location decision is often presented as a dichotomous choice, i.e., whether to live near or far away from a transit station. Proximity is measured by a circular buffer around a station, often with a half-mile radius. The extent of this buffer is usually justified on empirical grounds. Cervero (2007), for example, used a half-mile radius in estimating a nested logit model of the joint determination of mode and location. This measure of transit proximity fails to account for barriers that prevent access to a station that lies within the half-mile radius. Some researchers have considered residential location as a choice to reside within a geographical unit, such as a traffic assignment zone (Bhat and Guo 2004; Pinjari et al. 2007).

The use of transit proximity as a proxy for residential location, while dictated by the need to sort out the influence of the built environment from self-selection, is not based on any other theoretical underpinnings about the decision-making process that is at the heart of urban residential location theory. That is, it does not take into consideration the trade-off between housing and transportation costs that, at the margin, determine where an individual decides to locate. For example, the standard theory of location shows that individuals choose an optimal distance between work and home given housing and transportation costs. In a monocentric model that only looks at travel between home and the CBD, individuals locate at a distance where the marginal cost of transportation is equal to the marginal housing cost savings obtained by a move farther out from the CBD (Moses 1958; Muth 1969). Recent departures from this view consider that individuals can locate anywhere in an urban area, choosing an optimal home-work distance that optimizes also the amount of non-work travel and non-work activities (Anas and Kim 1996; Anas and Xu 1999). Further explorations also consider the role of trip chaining behavior (Anas 2007).

Activity space: spatial dispersion of non-work activities

The concept of activity space, although not new to behavioral sciences, is novel in terms of its application to travel behavior. The relationship between urban form and geographical patterns of activities has been studied only recently, due to the availability of specialized travel diary data and increasingly sophisticated geospatial tools. A growing field of research that looks at the relationship between urban form and the spatiotemporal allocation of activities and travel provides additional insight on the impact of the built environment. Recent research describing travel behavior and the influence of urban morphology and entire patterns of daily household activities and travel demonstrates how households residing in decentralized, lower density, urban areas tend to have a more dispersed activity-travel pattern then their counterpart residing in centralized, high density urban areas (Buliung and Kanaroglou 2006, 2007; Maoh and Kanaroglou 2007).

This study explicitly accounts for the influence of the built environment in affecting the spatial dispersion of activities and how spatial dispersion affects the demand for travel and location decisions. This effect is accounted for by introducing the variable *activity space*, *AS,* into the model. The extent of the activity space is assumed to be affected by the built environment. Densely populated urban areas tend to cluster activity locations together thus shrinking the size of the activity space. This affects the spatial allocation of activities, thus affecting the demand for travel. As seen in the next chapter, there exist several ways empirically to measure the spatial dispersion of activities.

Trip Chaining, *TC*

According to activity-based modeling practice, trip chaining describes how travelers link trips between locations around an activity pattern. In this context, a trip from home to work with an intermediate stop to drop children off at day care is an example of a trip chain. In the literature there is not a formal definition of *trip chain,* and different terms and expectations exist as to what kind of trips should be considered as part of a chain (McGuckin and Murakami 1999).

Sometimes, the term *trip chain* is used interchangeably with the term *tour* to indicate a series of trips that start and end at home.

In this study, it is hypothesized that trip chaining occurring on the home-job commuting pair saves time. These time savings in turn can be either allocated to additional non-work travel, thus increasing the overall demand for travel (e.g., total number of trips), or be used to determine a longer commute (i.e., a home-job commuting pair farther apart). The hypothesis of increased discretionary travel due to trip-chaining has recently been theoretically demonstrated (Anas 2007). The hypothesis of a positive relationship between more complex trip chains and the home-work commute is confirmed by empirical work. For example, in an analysis of trip chaining involving home-to-work and work-to-home trips using data from the 1995 nationwide personal transportation survey (NPTS), McGucking and Murakami (1999) found that people are more likely to stop on their way home from work, rather than on their way to work. About 33 percent of women linked trips on their way to work compared with 19 percent of men, while 61 percent of women and 46 percent of men linked trips on their way home from work. Using the 1991 Boston Household Travel Survey, Bhat (1997) found that about 38 percent of individuals made stops during the commute trip. Davidson (1991) found similar results from her analysis of commute behavior in a suburban setting, showing that travelers rely heavily on trip chaining in an urban context characterized by higher spatial dispersion of non-work activities. Other studies also provide empirical evidence of increased stop-making during the commute periods (Bhat 2001) or how the ability to link trips is enhanced by the flexibility inherent in automobile use (Strathman 1995).

Travel Demand, *TD*

Travel demand is herein treated as a derived demand brought about by the need to purchase goods and services. Travel demand, *TD*, measures the number of work and non-work transit trips at the household level. The decision process behind the choice of the number of trips, as formalized by this framework, considers trip generation as a function of trip chaining and exogenous residential location and socio-demographic factors. The constrained maximization problem of the joint determination of activity space and trip-chaining defines an optimal vector of non-work trips, given residential location and urban form characteristics (e.g., residential and employment density levels, land-use mix). This treatment of travel demand as derived from the desire to engage in out-of-home activities departs in terms of behavioral sophistication from the treatment of trip generation as developed by Boarnet and Crane (2001) in their analysis of travel demand and urban design. In Boarnet and Crane (2001) trip demand functions are either directly affected by land use or indirectly (by influencing the cost of travel).

In contrast, in this model land use (i.e., urban form) directly affects the spatial allocation of activities. It is the budget-constrained utility-maximization behavior that defines optimal travel patterns. The complexity of this mechanism is better shown in the ensuing comparative static analysis, which allows ascertaining the effect that urban form exerts on the demand for travel.

Comparative Static Analysis

The basic theoretical implications of Model I can be explored by employing comparative static analysis. This section considers the impact of changes in exogenous density, D and exogenous residential location, RL, on travel demand, *TD*. Basically, starting from an equilibrium state, the impacts of an increase in density and residential location on the initial equilibrium are determined. The objective is to see what happens to transit demand as density levels change.

The most relevant results of the comparative static analysis are summarized below, while the derivation of the comparative statics and the necessary assumptions to carry them out are a detailed in the appendix.

Effects of an increase in density, $\overline{D}$

The effect of an increase in density on travel demand is obtained as

$$\frac{dTD}{dD} = \frac{\overbrace{\overbrace{TD_{AS}}^{(-)}\overbrace{AS_D}^{(-)}}^{\alpha} + \overbrace{\overbrace{TD_{TC}}^{(+)}\overbrace{TC_{AS}}^{(+)}\overbrace{AS_D}^{(-)}}^{\beta}}{\underbrace{1-\underbrace{AS_{TC}}_{(+)}\underbrace{TC_{AS}}_{(+)}}_{(+)}} \gtrless 0 \tag{3.4}$$

where subscripts denote a partial differentiation of the subscripted variable with respect to the variable abbreviated by the subscript. The product $\alpha = TD_{AS}AS_{TD}$ gives the increase in transit demand caused by a contraction in the activity space as a result of increased density. The product $\beta = TD_{TC}TC_{AS}AS_D$ gives the decrease in transit demand caused by decreasing trip chaining as a result of the contraction of the activity spaces caused by increased density

Based on assumed relationship between spatial dispersion of activities and trip chaining, the result of this analysis shows that changes in density levels exert two contrasting effects on the demand for transit trips. The result shows an ambiguous effect of an increase in density on transit demand (as measured in total linked trips per household). Indeed, for ${dTD}/{dD} > 0$ it must be that $\alpha > -\beta$. In other words for transit demand to be positively related to density, the increase in transit demand caused by a contraction in the activity space (as a result of increased density, $\alpha > 0$) must be numerically greater than the reduction in transit demand caused by reduced trip chaining (as a result of increased density, $\beta < 0$).

This explanation is inherent in the determinants of trip chaining behavior. In higher density environments, as the spatial extent of non-work activities reduces, trip chaining needs decrease, but individual trips increase and individuals prefer to make non-chained trips. First, increased density reduces the activity space, which directly increases the demand for non-chained trips. Second, increased density reduces the activity space, which reduces the need to chain trips (as time-saving opportunities decrease) and thus the demand for transit trips. Therefore, an increase in density increases transit trips if and only if the direct effect of a contraction of the activity space is greater in magnitude than the indirect effect of the decrease in trip chaining. In other words, the above comparative static result shows that the increase in density exerts two opposite effects on transit demand.

Change in Residential Location, *RL*

Next, we derive the comparative statics of an increase in residential location, *RL*. Note that *RL* is considered as predetermined in Model I. This model is suited to either describe a situation where residential location is considered as predetermined, such as a short run time frame or can be used to cross compare decision making among households at any point in time. The question to be answered is: "What happens to transit demand as the job-residence pair changes?" Using cross sectional data, this question can be translated as: "How does transit demand differ for those households facing long commutes from those making short commutes?"

The comparative static result describing the impact of a change in residential location on the demand for transit trips is

$$\frac{dTD}{dRL} = \frac{\overbrace{TD_{RL}}^{(\pm)} + \overbrace{TC_{RL}}^{(+)}\overbrace{TD_{TC}}^{(+)} + \overbrace{AS_{TC}}^{(+)}\left(\overbrace{TC_{RL}}^{(+)}\overbrace{TD_{AS}}^{(-)} - \overbrace{TC_{AS}}^{(+)}\overbrace{TD_{RL}}^{(\pm)}\right)}{\underbrace{1 - \underbrace{AS_{TC}}_{(+)}\underbrace{TC_{AS}}_{(+)}}_{(+)}} \gtrless 0 \tag{3.5}$$

As previously discussed, an increase in residential location increases trip chaining ($TC_{RL} > 0$), which in turn positively affects both the size of the activity space, *AS*, and the demand for transit services. The overall effect on transit demand hinges on the sign of TD_{RL}. To the extent that an urban area is well served by transit, then the relationship between transit demand and residential location is positive. A positive relationship is observed in older, more monocentric cities, where existing transit services support commuting. On the other hand, if supply constraints exist, transit demand declines as the job-residence distance increases. Therefore, the overall effect on transit demand due to a change in location depends on both the sign and magnitude of TD_{RL}.

Model II: Endogenous Residential Location, Exogenous Density

In this model, we relax the assumption of exogenous residential location. Treated as a choice variable, residential location is the outcome of a trade-off between transportation and land-use costs. Taking into account idiosyncratic preferences for location, households choose an optimal home-work commute pair, while at the same time optimizing goods consumption and the ensuing non-work travel behavior (optimal non-work trip chaining and activity space). This model is specified as

$$TC = TC(AS, RL, WD, X_{TC}) \tag{3.8}$$

$$AS = AS(TC, \overline{D}, X_{AS}) \tag{3.9}$$

$$TD = TD(TC, AS, RL, WD, X_{TD}) \tag{3.10}$$

$$RL = RL(TC, TD, X_{RL}) \tag{3.11}$$

where X_{RL} is a vector of controls specific to the RL equation and all other variables are as defined earlier.

Comparative Static Analysis

The complete comparative statics are presented in Appendix B. A discussion of the findings is presented below. Note that the inclusion of the endogenous residential location equation, *RL*, complicates the computation of the total partial derivatives.

Effects of an Increase in Density, *D*

The effect of an increase in density on travel demand is obtained as

$$\frac{dTD}{dD}=\frac{\overbrace{AS_D}^{(-)}\left[\left(1-\overbrace{RL_{TC}}^{(+)}\overbrace{TC_{RL}}^{(+)}\right)\overbrace{TD_{AS}}^{(-)}+\overbrace{TC_{AS}}^{(+)}\left(\overbrace{RL_{TC}}^{(+)}\overbrace{TD_{RL}}^{(\pm)}+\overbrace{TD_{TC}}^{(+)}\right)\right]}{\underbrace{1-\underbrace{RL_{TC}}_{(+)}\underbrace{TC_{RL}}_{(+)}-\underbrace{RL_{TD}}_{\pm}\underbrace{TD_{RL}}_{\pm}+\underbrace{AS_{TC}}_{+}\left[-\underbrace{RL_{TD}}_{\pm}\underbrace{TC_{RL}}_{+}\underbrace{TD_{AS}}_{-}+\underbrace{TC_{AS}}_{+}\left(-1+\underbrace{RL_{TD}}_{\pm}\underbrace{TD_{RL}}_{\pm}\right)\right]-\underbrace{RL_{TD}}_{\pm}\underbrace{TC_{RL}}_{+}\underbrace{TD_{TC}}_{+}}_{|J|}}\lesseqgtr 0 \qquad (3.12)$$

In the long run, the activity space, transit demand, trip chaining and residential location are all jointly determined. Exogenous changes in density levels therefore affect all these variables. An increase in density directly contracts the activity space, whereas it indirectly reduces trip chaining and ambiguously affects transit demand through its effect on the activity space. The effect on residential location operates through the effect on transit demand, but that effect is ambiguous. This renders the effect of density on transit demand ambiguous as well. Comparing equation (3.12) to equation (3.4), we see that the complexity of the relationship between transit demand and density increases substantially.

Model III: Endogenous Residential Location, Endogenous Density

In this last extension to Model I, the assumption of exogenous density is relaxed. This model translates the conceptual framework of Figure 3.1 into the following analytical model

$$TC = TC(AS, RL,\ WD, X_{TC}) \qquad (3.13)$$

$$AS = AS(TC, D, X_{AS}) \qquad (3.12)$$

$$TD = TD(TC, AS, RL, WD, X_{TD}) \qquad (3.13)$$

$$D = D(RL, AS, X_D) \tag{3.14}$$

$$RL = RL(TC, , X_{RL}) \tag{3.15}$$

In the long run, the simultaneous choice of location and travel decisions are assumed to affect density levels across a given urban area. This model best describes a long-run equilibrium, in which both location and travel decisions are optimized under constraint. Urban form is treated as endogenous to the process and is itself affected by household travel decisions and location behavior. Aspects of this relationship and its influences on transit patronage have been previously considered in the literature. For example, while modeling long-run transit demand responses to fare changes, Voith (1997) treats density as endogenous and being affected directly by transit patronage levels. In the long run, these levels are affected by supply-side changes. Voith (1997) assumes that as transit services improve, more people tend to live in proximity to transit stations, thus increasing the demand for transit services. Empirically, the latter can be acknowledged by treating transit station proximity as endogenous to this process.

Ideally, empirical testing of this model would rely on panel data of individual travel diaries. Generally, however, panel data are unavailable and cross-section data are relied on. With cross section data, we can study changes in behavior by controlling for individual heterogeneity.

Comparative-Static Analysis

Given the endogenous treatment of density, we can use this model to test the effects of policies geared at directly affecting density, such as policy interventions intended to increase density around transit stations. Assuming an exogenous shock, θ, positively affecting density, comparative statics can be obtained. The inclusion of two more equations complicates the calculations to derive the relevant comparative static results. The results are basically the same as Model II, although the expected magnitudes of impacts differ. To avoid cluttering the text,

Appendix B reports the comparative statics results, which we will use in the empirical work of Chapter 4. Table 3.1 reports a summary of the comparative statics highlighting the expected signs from changes in the most relevant variables affecting transit demand.

TABLE 3.1 Comparative Static Results

Exogenous Variable	*D*	*RL*	AS_{φ}^{*}	*WD*
Effect on TD	+/-	+/-	-	-

**Shift parameters affecting AS*

Empirical Findings

We empirically tested the models employing travel diary data from the 2000 Bay Area Travel Survey (BATS 2000). Theoretical and empirical findings provide evidence of a significant causal influence of land-use patterns on transit patronage, which, in turn, affects consumption and non-work travel. It is found that gross population density does not have a large impact on transit demand and that the relative magnitude of the effect decreases when residential location is treated as endogenous. A 20-percent increase in gross population density (or an increase of about 1,830 persons per square mile) increases transit demand by 4.8 to 9.5 percent.

The relevance of land-use policies geared at influencing transit patronage by providing a mix of residential and commercial uses is highlighted by the elasticity of travel demand with respect to changes in retail establishment density. Results show that a 20-percent increase in retail establishment density (or an increase of about 28 establishments per square mile) increases transit demand by three percent.

Households living farther from work, *ceteris paribus*, use less transit. It is shown that trip-chaining behavior explains this observation. Households living far from work engage in complex trip chains and have, on average, a more dispersed activity space, which requires reliance on more flexible modes of transportation. Therefore, reducing the spatial allocation of activities and improving transit accessibility at and around subcenters would increase transit demand. Similar effects can be obtained by increasing the presence of retail locations in proximity to transit-oriented households.

An established central business district (CBD) is still a relevant driver of transit use, as highlighted by an elasticity of transit demand with respect to distance to the CBD of –1.17. Subcenters also play a role, indicating the need to provide services to decentralized areas to increase ridership.

Transit-oriented development has a positive and statistically significant impact on transit use. The presence of a TOD station at the point of origin increases transit demand by about 21 percent. The presence of a transit station in proximity to a workplace also has a significant positive impact on ridership, as indicated by the magnitude of the elasticity across all three models.

Notwithstanding the validity of endogeneity tests performed in the empirical analysis, there still exists the possibility that some of the variables treated as exogenous are, in fact, endogenous. For example, while this study treats vehicle ownership as exogenous and not directly influenced by the location decision, the literature review encountered studies that considered vehicle ownership endogenous to the residential location process and to density levels. One extension to the research would be to include an endogenous treatment of this and other mode-choice variables. Additional extensions would account for the joint allocation of time use among household members.

Appendix

Comparative-Static Analysis

In this appendix, we derive the most relevant comparative statics results of Model I through Model III. In Model I, we consider the impact of changes in exogenous density, $\overline{D}$, and exogenous residential location, $\overline{RL}$, on travel demand, *TD*. Starting from an equilibrium state, we consider the impact of an increase in density and residential location on the initial equilibrium. The objective is to see what happens to transit demand as density levels change. To conduct comparative-static analysis, a set of basic assumptions related to trip chaining behavior, activity space, and urban form must first be introduced. Also, although trips are integers in reality, they are herein treated as a continuous non-negative variable for analytical purposes.

Assumption A.1

Residential location is defined as the optimal *job-residence pair* in an urban area in which jobs and residences are dispersed. Following urban residential location theory, the location decision is assumed to be the result of a trade-off between housing expenditures and transportation costs, given income and the mode-choice set. Following Anas and his associates (Anas and Kim 1996; Anas and Xu 1999), the location decision is also based on idiosyncratic preferences for location and travel. As the distance between this two locations increases, the need to engage in trip-chaining also increases. Trip chaining, as shown in Anas (2007), allows saving time, which, in turn can be allocated either to a farther move away from work (more commute time), to be spent as leisure time, or to be used for more non-work travel. As the distance defining the *job-residence pair* increases, then the need to chain non-work trips increases

$$TC_{RL} = \frac{\partial TC}{\partial RL} > 0 \tag{a.1}$$

But this happens at a decreasing rate

$$\frac{\partial^2 TC}{\partial RL^2} < 0 \tag{a.2}$$

Assumption A.2

If density, $\overline{D}$, increases, then non-work activity locations, such as shopping or recreational locations, tend to be more clustered together, thus reducing the household activity area

$$AS_D = \frac{\partial AS}{\partial D} < 0 \tag{a.3}$$

Assumption A.3

If the household activity space gets more dispersed (increases) then trip chaining increases:

$$TC_{AS} = \frac{\partial TC}{\partial AS} > 0 \tag{a.4}$$

with

$$\frac{\partial^2 TC}{\partial AS^2} < 0 \tag{a.5}$$

This is reciprocal to *A.2*. As the household activity space grows or gets more dispersed the need to engage in trip chaining increases. Assumptions A.2 and A.3 are based on findings from generalizing the partial equilibrium model of trip chaining developed by Anas (2007). We report this analysis in a separate appendix available upon request.

Empirical evidence that confirms this hypothesis is found in Thomas and Noland (2006) who, in a multivariate analysis of trip chaining behavior, find a positive relationship between lower densities and the complexity of trip chaining behavior. Noland and Thomas found that density environments lead to both a greater reliance upon trip chaining and tours that involve more stops.

Assumption A.4

As trip chaining increases, the household activity space increases:

$$AS_{TC} = \frac{\partial AS}{\partial TC} > 0 \tag{a.6}$$

This assumption means that factors that directly affect the trip chaining function, *TC*, result in feedback effects on activity space, *AS*. These feedback effects are less intense and marginally decreasing

$$\frac{\partial^2 AS}{\partial TC^2} < 0 \tag{a.7}$$

Model I Comparative-static Results

Now, consider Model I. Equations (3.1), (3.2), and (3.3) can be written as specific functions in the form $F^j(TC, AS, RL, D, X_{TC}, X_{AS}, X_{TT})$. With continuous partial derivatives and with the relevant assumptions A.3 and A.4, the following nonzero *Jacobian determinant*[1] is obtained

$$|J| = \begin{vmatrix} \frac{\partial F^1}{\partial TC} & \frac{\partial F^1}{\partial AS} & \frac{\partial F^1}{\partial TD} \\ \frac{\partial F^2}{\partial TC} & \frac{\partial F^2}{\partial AS} & \frac{\partial F^2}{\partial TD} \\ \frac{\partial F^3}{\partial TC} & \frac{\partial F^3}{\partial AS} & \frac{\partial F^3}{\partial TD} \end{vmatrix} = \begin{vmatrix} 1 & -TC_{AS} & 0 \\ -AS_{TC} & 1 & 0 \\ -TD_{TC} & -TD_{AS} & 1 \end{vmatrix} = 1 - \overbrace{AS_{TC}}^{(+)} \overbrace{TC_{AS}}^{(+)} \neq 0 \tag{a.8}$$

Therefore, *TC*, and *AS* can be considered implicit functions of (RL, X_{TC}, D, X_{AS}) at and around any point that satifies Equations (1) and (2), which would then be an equilibrium solution, $\overline{TC}$, $\overline{AS}$, and $\overline{TD}$. Hence the implicit function theorem justifies writing

$$\overline{TC} = f^1(RL, D, WD, X_{TC}, X_{AS}, X_{TD}) \quad \text{(a.9)}$$

[1] The *Jacobian determinant* (or a *Jacobian*, for short), is a determinant of a matrix of partial derivatives, which tests functional dependence among a set of functions. Given the equation system, partial derivatives needed for comparative-static analysis (see previous footnote) can be obtained if the Jacobian, *J*, is non-zero.

$$\overline{HAS} = f^2(RL, D, WD, X_{TC}, X_{AS}, X_{TD}) \tag{a.10}$$

$$\overline{TD} = f^3(RL, D, WD, X_{TC}, X_{AS}, X_{TD}) \tag{a.11}$$

indicating that the equilibrium values of the endogenous variables are implicit functions of the exogenous variables and parameters. The partial derivatives of the implicit functions are in the nature of comparative-static derivatives. To find these, the partial derivative of the *F* functions, evaluated at the equilibrium state of the model, are needed.

Next, the comparative-static analysis is conducted to ascertain the effect of changes brought about by changes in density, residential location and transit station proximity (i.e., changes in walking distance)

Effects of an Increase in Density, $\overline{\boldsymbol{D}}$

The general form for the comparative-static analysis of Model I is given by

$$\begin{vmatrix} 1 & -TC_{AS} & 0 \\ -AS_{TC} & 1 & 0 \\ -TD_{TC} & -TD_{AS} & 1 \end{vmatrix} \begin{bmatrix} \partial\overline{TC}/\partial D \\ \partial\overline{AS}/\partial D \\ \partial\overline{TD}/\partial D \end{bmatrix} = \begin{bmatrix} 0 \\ AS_D \\ 0 \end{bmatrix} \tag{a.12}$$

Density Effect on Trip Chaining

First, the effect of increased density on trip chaining is considered. By applying Cramer's rule,[2] the total partial derivative is computed as

$$d\overline{TC}/dD = \frac{\begin{vmatrix} 0 & -TC_{AS} & 0 \\ AS_D & 1 & 0 \\ 0 & -TD_{AS} & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{\overset{+}{\overbrace{TC_{AS}}}\overset{-}{\overbrace{AS_D}}}^{(-)}}{\underbrace{1-\underbrace{AS_{TC}}_{(+)}\underbrace{TC_{AS}}_{(+)}}_{(+)}} < 0 \tag{a.13}$$

[2] *Cramer's rule* is a method of matrix inversion that enables a convenient, practical way of solving a linear-equation system.

The results of this comparative-static show that an increase in density causes a clustering of activities which contracts the activity space, which, in turn, reduces the need to engage in trip chaining. The total reduction in trip chaining also accounts for the feedbacks into Equation (1) coming from Equation (2) by way of TC_{AS}. This outcome has been confirmed in the literature on trip chaining behavior, which shows that lower density environments increase the need to engage in trip chaining (Noland and Thomas 2007; Wallace, Barnes, and Rutherford 2000).

Density Effect on Activity Space

The effect of an increase in *D* on trip chaining is obtained in the same manner

$$d\overline{AS}/_{dD} = \frac{\begin{vmatrix} 1 & 0 & 0 \\ -AS_{TC} & AS_D & 0 \\ -TD_{TC} & 0 & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{AS_D}^{(-)}}{(+)} < 0 \qquad \text{(a.14)}$$

Note, by assumption A.*1*, we have that $\partial\overline{AS}/_{\partial D} < 0$. Therefore, an increase in density contracts the activity space both directly and indirectly through feedback effect coming by way of the Equation (1) ($AS_{TC}TC_{AS}$).

Effect on Transit Demand

The effect of an increase in density on transit demand, the most relevant comparative-static analysis in the context of this study is obtained as

$$\frac{d\overline{TD}}{d\overline{D}} = \frac{\begin{vmatrix} 1 & -TC_{AS} & 0 \\ -AS_{TC} & 1 & AS_D \\ -TD_{TC} & -TD_{AS} & 0 \end{vmatrix}}{|J|} = \frac{\overbrace{\overbrace{TD_{AS}}^{(-)}\overbrace{AS_D}^{(-)}}^{\overset{(+)}{\alpha}} + \overbrace{\overbrace{TD_{TC}}^{(+)}\overbrace{TC_{AS}}^{(+)}\overbrace{AS_D}^{(-)}}^{\overset{(-)}{\beta}}}{|J|} \gtrless 0 \qquad \text{(a.14)}$$

where

α = change in transit demand caused by a contraction in activity space as a result of increased density

β= change in transit demand caused by decreasing trip chaining as a result of increased density

The result shows an ambiguous effect of density on transit demand (as measured in total linked trips per household). Indeed, for $\frac{d\overline{TD}}{d\overline{D}} > 0$ it must be that $\alpha > -\beta$. In other words for transit demand to be positively related to density it must be that the increase in transit demand caused by a contraction in activity space (as a result of increased density, $\alpha > 0$) is greater than the reduction in transit demand caused by reduced trip chaining (as a result of increased density, $\beta < 0$).

This explanation is inherent to the determinants of trip chaining behavior. In higher density environments, as the spatial extent of non-work activities reduces, trip chaining needs decrease but individual trips increase and individuals prefer to make non-chained trips. First, increased density reduces the extent of the activity space, which directly increases the demand for non-chained transit trips. Second, higher densities reduce the activity space, which reduces the need to chain trips (as time savings opportunities decrease) and thus the demand for transit trips. Thus an increase transit trips occurs if transit demand is more sensitive to changes affecting the spatial allocation of non-work activities than to changes affecting trip chaining behavior. In other words, the above comparative-statics result shows that the increase in density exerts two opposite effects on transit demand. This result relies on two additional assumptions, namely

$$\frac{\partial TD}{\partial TC} > 0 \tag{a.15}$$

The demand for transit trips increases as trip chaining increases. An increase in the number of chained trips overall increases the demand for transit (or any other mode). This is brought about by the initial model specification which presents a transit trip demand as a function of trip chaining.

$$\frac{\partial TD}{\partial AS} < 0 \tag{a.16}$$

This means that increased spatial dispersion of non-work activities cannot be accommodated by additional transit trips. Given the characteristics of transit service supply (being fixed at least in the short to medium run), increased spatial dispersion is accommodated by substituting transit travel with other, more flexible, modes, such as auto travel. The latter is more flexible in terms of allowing serving a more dispersed activity space. This assumption confirms that transit and auto are non-perfect substitutes.

Living Farther from a Transit Station (change in walking distance, WD)

Next, the comparative-statics of an increase in walking distance, *WD*, are derived. Next, the effect of an increase in distance from the nearest transit station is considered. The empirical literature provides unequivocal evidence of a negative relationship between distance to transit stops and the demand for transit services (Cervero 2007; Cervero and Kockelman 1997). The debate is mostly centered on the magnitude of this relationship, as highlighted by the growing body of literature on residential self-selection. All else equal, being located farther away from a transit station results in reduced transit demand

$$\frac{d\overline{TD}}{d\overline{WD}} = \frac{\begin{vmatrix} 1 & -TC_{AS} & TC_{WD} \\ -AS_{TC} & 1 & 0 \\ -TD_{TC} & -TD_{AS} & TD_{WD} \end{vmatrix}}{|J|} = \frac{\overbrace{\overbrace{TD_{WD}}^{(-)}\left(1+\overbrace{TD_{TC}}^{(+)}\right)}^{(-)}+\overbrace{\overbrace{AS_{TC}}^{(+)}\left(\overbrace{TC_{WD}}^{(-)}\overbrace{TD_{AS}}^{(-)}-\overbrace{TC_{AS}}^{(+)}\overbrace{TD_{WD}}^{(-)}\right)}^{(+)}}{|J|} < 0 \qquad \text{(a.17)}$$

A Move Farther Away from Work (change in residential location)

Next the comparative-statics of an increase in residential location, *RL,* are derived. Note that *RL* is considered as exogenous in Model I, to indicate short run equilibrium. The resulting comparative-static follows

$$\begin{vmatrix} 1 & -TC_{AS} & 0 \\ -AS_{TC} & 1 & 0 \\ -TD_{TC} & -TD_{AS} & 1 \end{vmatrix} \begin{bmatrix} \partial\overline{TC}/\partial RL \\ \partial\overline{AS}/\partial RL \\ \partial\overline{TD}/\partial RL \end{bmatrix} = \begin{bmatrix} TC_{RL} \\ 0 \\ TD_{RL} \end{bmatrix} \tag{a.18}$$

Effect on Trip Chaining, TC

First, the effect of a move farther away from work on trip chaining is considered. By Assumption A.1, this has a positive impact on trip chaining. The new equilibrium results in a higher number of trips per chain. When testing this hypothesis empirically and using cross-sectional data, individuals with a more extended commute are expected to engage in a higher number of trips per chain (or in more complex tours characterized by more stops).

$$d\overline{TC}/dRL = \frac{\begin{vmatrix} TC_{RL} & -TC_{AS} & 0 \\ 0 & 1 & 0 \\ TD_{RL} & -TD_{AS} & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{TC_{RL}}^{(+)}}{|J|} > 0 \tag{a.19}$$

In a longitudinal context, a move farther out entails more distance and more time spent commuting, which increases the propensity to engage in trip chaining to save overall time.

Effect on Activity Space,AS

The effect of an increase in *RL* on the activity space is given by

$$d\overline{AS}/dRL = \frac{\begin{vmatrix} 1 & TC_{RL} & 0 \\ -AS_{TC} & 0 & 0 \\ -TD_{TC} & TD_{RL} & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{\overbrace{AS_{TC}}^{(+)}\overbrace{TC_{RL}}^{(+)}}^{(+)}}{(+)} > 0 \tag{a.20}$$

A move farther away from work increases trip chaining, which in turn increases the activity space. This increase is indirect as it comes by way of Equation (a.9). The empirical work will reveal information on its magnitude.

Effect on Transit Demand, TD

The change in transit demand caused by a change in residential location is given by:

$$d\overline{TD}\Big/_{dRL} = \frac{\begin{vmatrix} 1 & -TC_{AS} & TC_{RL} \\ -AS_{TC} & 0 & 0 \\ -TD_{TC} & -TD_{AS} & TD_{RL} \end{vmatrix}}{|J|} = \frac{\overbrace{TD_{RL}}^{(\pm)}+\overbrace{AS_{TC}}^{(+)}\left(\overbrace{TC_{RL}}^{(+)}\overbrace{TD_{AS}}^{(-)}-\overbrace{TC_{AS}}^{(+)}\overbrace{TD_{RL}}^{(\pm)}\right)+\overbrace{TC_{RL}}^{(+)}\overbrace{TD_{TC}}^{(+)}}{(+)} \gtrless 0 \quad \text{(a.21)}$$

The overall effect on transit demand hinges on the sign of TD_{RL}. To the extent that an urban area is well served by transit, then the relationship between transit demand and residential location is positive. A positive relationship is observed in older, more monocentric-type cities, with existing transit services supporting major work commute travel routes. On the other hand, if supply constraints exist, transit demand declines as the job-residence distance increases. Therefore, the overall effect on transit demand due to a change in location depends on both the sign and magnitude of $TD_{RL}(TD_{RL} \lesseqgtr 0)$.

Exogenous shift in Trip Chaining, TC_φ

This comparative-static allows computing the total derivative of transit demand with respect to exogenous changes directly affecting only the trip chaining equation. Let φ be a change in an exogenous variable appearing only in the trip chaining equation, then a change in φ has the following effect on transit demand

$$d\overline{TD}\Big/_{dTC_\phi} = \frac{\begin{vmatrix} 1 & -TC_{AS} & TC_{RL} \\ -AS_{TC} & 0 & 0 \\ -TD_{TC} & -TD_{AS} & TD_{RL} \end{vmatrix}}{|J|} = \frac{\overbrace{TC_\varphi}^{(\pm)}\left(\overbrace{TD_{TC}}^{(+)}+\overbrace{AS_{TC}}^{(+)}\overbrace{TD_{AS}}^{(-)}\right)}{|J|} \quad \text{(a.22)}$$

This result, for example is used to assess the effect a change in distance to the nearest subcenter under the empirical specification of equation (4.5) in Chapter 4.

Exogenous shift in Activity Space, AS_φ

This comparative-static allows computing the total derivative of transit demand with respect to exogenous changes directly affecting only the activity space equation. Let φ be a change in an exogenous variable appearing only in the activity space equation, then a change in φ has the following effect on transit demand

$$d\overline{TD}\Big/dAS_\varphi = \frac{\begin{vmatrix} 1 & -TC_{AS} & TC_{RL} \\ -AS_{TC} & 0 & 0 \\ -TD_{TC} & -TD_{AS} & TD_{RL} \end{vmatrix}}{|J|} = \frac{AS_\varphi\left(\overbrace{TD_{AS}}^{(-)}+\overbrace{TC_{AS}}^{(+)}\overbrace{TD_{TC}}^{(+)}\right)}{|J|} \tag{a.23}$$

This result is used to evaluate the effect of a change in retail establishment density, as appearing in equation (4.6) on transit demand.

Model II

In this model, the assumption of residential location is relaxed. Treated as a choice variable, residential location is the outcome of a trade-off between transportation and land use costs. Taking into account idiosyncratic preferences for location, households choose an optimal home-work commute pair, while at the same time optimizing goods consumption and the ensuing non-work travel behavior (optimal non-work trip chaining and activity space). The specification of the model is given by equations (3.8) to (3.11). The inclusion equation (3.11) requires deriving a new Jacobian determinant

$$|J| = \begin{vmatrix} 1 & -TC_{AS} & 0 & -TC_{RL} \\ -AS_{TC} & 1 & 0 & 0 \\ -TD_{TC} & -TD_{AS} & 1 & -TD_{RL} \\ -RL_{TC} & -RL_{TD} & 0 & 1 \end{vmatrix} = 1 - \overbrace{RL_{TC}}^{(+)}\,\overbrace{TC_{RL}}^{(+)} - \overbrace{AS_{TC}}^{(+)}\left(\overbrace{TC_{AS}}^{(+)} + \overbrace{RL_{TD}}^{(\pm)}\,\overbrace{TC_{RL}}^{(+)}\right) \tag{a.24}$$

Given the complexity introduced by adding equation (a.24) to the model, the following comparative-static analysis focuses only on impacts of changes affecting the demand for transit trips.

Effect of an Increase in Density on Transit Demand, *D*

As shown by this result, the ultimate effect on transit demand when exogenous density levels change is relatively larger than model I. Using Cramer's rule, the change is computed as

$$\frac{d\overline{TD}}{d\overline{D}} = \frac{\begin{vmatrix} 1 & -TC_{AS} & 0 & -TC_{RL} \\ -AS_{TC} & 1 & AS_D & 0 \\ -TD_{TC} & -TD_{AS} & 0 & -TD_{RL} \\ -RL_{TC} & -RL_{TD} & 0 & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{AS_D}^{(-)}\left[\left(1-\overbrace{RL_{TC}}^{(+)}\overbrace{TC_{RL}}^{(+)}\right)\overbrace{TD_{AS}}^{(-)}+\overbrace{TC_{AS}}^{(+)}\left(\overbrace{RL_{TC}}^{(+)}\overbrace{TD_{RL}}^{(\pm)}+\overbrace{TD_{TC}}^{(+)}\right)+\overbrace{RL_{TD}}^{(\pm)}\left(\overbrace{TD_{RL}}^{(\pm)}+\overbrace{RL_{TD}}^{(\pm)}\overbrace{TD_{TC}}^{(+)}\right)\right]}{|J|} \qquad \text{(a.25)}$$

In the long run, the spatial extent of non-work activities, trip chaining and residential location are all jointly determined. Exogenous shifts in density levels affect this decision making process. An increase in density directly impacts the spatial extent of non-work activity locations in terms of an increased activity space, *AS* (i.e., activities are more disperse across the urban landscape). This increase affects trip chaining with feedback effects both on the demand for transit trips and residential location patterns in a looping fashion.

Effect of an Increase in Walking Distance, *WD*

A change in density affects both transit station and activity space directly and indirectly, as specified by

$$\frac{dTD}{d\overline{WD}} = \frac{\begin{vmatrix} 1 & -TC_{AS} & TC_{WD} & -TC_{RL} \\ -AS_{TC} & 1 & 0 & 0 \\ -TD_{TC} & -TD_{AS} & 0 & -TD_{RL} \\ -RL_{TC} & -RL_{TD} & TD_{WD} & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{\overbrace{TC_{WD}}^{(-)}\overbrace{TD_{TC}}^{(+)}}^{(-)}+\overbrace{TD_{WD}}^{(-)}+\overbrace{RL_{TC}}^{(+)}\left(\overbrace{\overbrace{TC_{WD}}^{(-)}\overbrace{TD_{RL}}^{(\pm)}}^{(\pm)}-\overbrace{\overbrace{TC_{RL}}^{(+)}\overbrace{TD_{WD}}^{(-)}}^{(-)}\right)+\overbrace{AS_{TC}}^{(+)}\left(\overbrace{TC_{WD}}^{(-)}\left(\overbrace{\overbrace{TD_{AS}}^{(-)}+\overbrace{RL_{TD}}^{(\pm)}\overbrace{TD_{RL}}^{(\pm)}}^{(\pm)}\right)-\left(\overbrace{\overbrace{TC_{AS}}^{(+)}+\overbrace{RL_{TD}}^{(\pm)}\overbrace{TC_{RL}}^{(+)}}^{(\pm)}\right)\overbrace{TD_{WD}}^{(-)}\right)}{\underbrace{1-\overbrace{\underbrace{AS_{TC}}_{(+)}\underbrace{TC_{AS}}_{(+)}}^{(+)}-\left(\underbrace{AS_{TC}}_{+}\right)\left(\overbrace{\underbrace{TC_{AS}}_{(+)}+\underbrace{RL_{TD}}_{(\pm)}\underbrace{TC_{RL}}_{(+)}}^{(\pm)}\right)}_{J}} \text{(a.26)}$$

The direct effect of density on transit proximity is due to two separate causes. First, higher densities improve transit proximity by reducing average walking distance to the nearest station. This can be identified as a supply side effect, in that more stations are likely to be located at higher densities. Second, at any given home-work commute pair arrangement individuals are more likely to utilize transit services to engage in both work and non-work activities. A change in density also indirectly affects transit station proximity, given its treatment as a choice variable. An increase in density results in more accessible non-work activities which reduce the need to engage in trip chaining, thus decreasing transit patronage. This in turn reduces the need to be located closely to a transit station. This indirect is expected to be of a smaller magnitude than the direct effect, allowing concluding that ψ is greater than zero.

On the other hand, the effect of density on transit demand by way of affecting the spatial extent of the activity space is the same as the one assessed previously under Model I specification.

Exogenous shift in Trip Chaining, TC_φ

En exogenous shift, φ, has the following impact on the demand for transit

$$\frac{d\overline{TD}}{dTC_\varphi} = \frac{\begin{vmatrix} 1 & -TC_{AS} & TC_\varphi & -TC_{RL} \\ -AS_{TC} & 1 & 0 & 0 \\ -TD_{TC} & -TD_{AS} & 0 & -TD_{RL} \\ -RL_{TC} & -RL_{TD} & 0 & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{TC_\varphi}^{(\pm)}\left(\overbrace{AS_{TC}}^{(+)}\overbrace{TD_{AS}}^{(+)}+\overbrace{RL_{TC}}^{(+)}\overbrace{TD_{RL}}^{(\pm)}+\overbrace{TD_{TC}}^{(\pm)}\right)}{|J|} \tag{a.27}$$

Exogenous shift in Activity Space, AS_φ

An exogenous change, φ, affecting *AS*, has the following effect on transit demand

$$\frac{d\overline{TD}}{dAS_\varphi} = \frac{\begin{vmatrix} 1 & -TC_{AS} & 0 & -TC_{RL} \\ -AS_{TC} & 1 & AS_\varphi & 0 \\ -TD_{TC} & -TD_{AS} & 0 & -TD_{RL} \\ -RL_{TC} & -RL_{TD} & 0 & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{AS_\varphi}^{(\pm)}\left[\left(1-\overbrace{RL_{TC}}^{(+)}\overbrace{TC_{RL}}^{(+)}\right)\overbrace{TD_{AS}}^{(-)}+\overbrace{TC_{AS}}^{(+)}\left(\overbrace{RL_{TC}}^{(+)}\overbrace{TD_{RL}}^{(\pm)}+\overbrace{TD_{TC}}^{(+)}\right)\right]}{|J|} \tag{a.28}$$

Exogenous shift in Residential Location, RL_φ

An exogenous change, φ, affecting residential location decisions, has the following impact on transit demand

$$\frac{d\overline{TD}}{dRL_\varphi} = \frac{\begin{vmatrix} 1 & -TC_{AS} & 0 & -TC_{RL} \\ -AS_{TC} & 1 & 0 & 0 \\ -TD_{TC} & -TD_{AS} & 0 & -TD_{RL} \\ -RL_{TC} & -RL_{TD} & RL_\varphi & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{RL_\varphi}^{(\pm)}\left[\overbrace{TD_{RL}}^{(\pm)}+\overbrace{AS_{TC}}^{(+)}\left(\overbrace{TC_{RL}}^{(+)}\overbrace{TD_{AS}}^{(-)}-\overbrace{TC_{AS}}^{(+)}\overbrace{TD_{RL}}^{(\pm)}\right)+\overbrace{TC_{RL}}^{(+)}\overbrace{TD_{TC}}^{(+)}\right]}{|J|} \tag{a.29}$$

Model III

In this model, the assumption of density as being exogenous to the model is relaxed. At any given home-commute pair arrangement, the decision to locate in proximity of a station is dependent upon transit patronage levels and factors related to density. Density in proximity to transit stations is affected by patronage levels. This relationship is described by equations (3.13) to (3.15). The computation of the Jacobian is further complicated by the addition of the density equation and is equal to

$$J = \begin{vmatrix} 1 & -TC_{AS} & 0 & -TC_{RL} & 0 \\ -AS_{TC} & 1 & 0 & 0 & -AS_D \\ -TD_{TC} & -TD_{AS} & 1 & -TD_{RL} & 0 \\ -RL_{TC} & 0 & -RL_{TD} & 1 & 0 \\ 0 & -D_{AS} & 0 & -D_{RL} & 1 \end{vmatrix} \tag{a.30}$$

$$|J| = 1 - \overbrace{RL_{TC}}^{(+)}\overbrace{TC_{RL}}^{(+)} - \overbrace{RL_{TD}}^{(\pm)}\overbrace{TD_{RL}}^{(\pm)} - \overbrace{AS_{TC}}^{(+)}\Bigg\{\left[\overbrace{RL_{TD}}^{(\pm)}\overbrace{TC_{RL}}^{(+)}\overbrace{TD_{AS}}^{(-)} + \overbrace{TC_{AS}}^{(+)}\left(1 - \overbrace{RL_{TD}}^{(\pm)}\overbrace{TD_{RL}}^{(\pm)}\right)\right] - \overbrace{RL_{TD}}^{(\pm)}\overbrace{TC_{RL}}^{(+)}\overbrace{TD_{TC}}^{(+)} +$$

$$\overbrace{AS_D}^{(-)}\left[-\overbrace{D_{RL}}^{(-)}\left(\overbrace{RL_{TC}}^{(+)}\overbrace{TC_{AS}}^{(+)} + \overbrace{RL_{TD}}^{(\pm)}\left(\overbrace{TD_{AS}}^{(-)} + \overbrace{TC_{AS}}^{(+)}\overbrace{TD_{TC}}^{(+)}\right)\right)\right] +$$

$$\overbrace{D_{AS}}^{(-)}\left[-1 + \overbrace{RL_{TC}}^{(+)}\overbrace{TC_{RL}}^{(+)} + \overbrace{RL_{TD}}^{(\pm)}\left(\overbrace{TD_{RL}}^{(\pm)} + \overbrace{TC_{RL}}^{(+)}\overbrace{TD_{TC}}^{(+)}\right)\right]\Bigg\} \neq 0$$

This model best describes long term equilibrium, where both location and travel decisions are optimized under constraint. Ideally, empirical testing of this model would rely on disaggregate travel diary data in the form of a panel that collects behavior of a same set of individuals across time. When dealing with observational data across different individuals at a point in time (i.e., a cross-sectional dataset), changes in behavior can be studied by controlling for individual heterogeneity.

The comparative-static analysis focuses on changes affecting the demand for transit trips.

Effect of Exogenous Shift in Density

Given the endogenous treatment of density, this model can be used to test the effects of policies geared at directly affecting density, such as policies that are intended to increase density around transit stations. Assuming an exogenous shock, θ, positively affecting density the following comparative-statics is obtained

$$\frac{d\overline{TD}}{dD_{\varphi}} = \frac{\begin{vmatrix} 1 & -TC_{AS} & 0 & -TC_{RL} & 0 \\ -AS_{TC} & 1 & 0 & 0 & -AS_{D} \\ -TD_{TC} & -TD_{AS} & 0 & -TD_{RL} & 0 \\ -RL_{TC} & 0 & 0 & 1 & 0 \\ 0 & -D_{AS} & D_{\varphi} & -D_{RL} & 1 \end{vmatrix}}{|J|} = \frac{\overbrace{RL_{\varphi}}^{(\pm)}\left\{1-\overbrace{AS_{TC}}^{(+)}\overbrace{TC_{AS}}^{(+)}-\overbrace{RL_{TC}}^{(+)}\overbrace{TC_{RL}}^{(+)}+\overbrace{AS_{D}}^{(-)}\left[-\overbrace{D_{RL}}^{(-)}\overbrace{RL_{TC}}^{(+)}\overbrace{TC_{AS}}^{(+)}+\overbrace{D_{AS}}^{(-)}\left(-1+\overbrace{RL_{TC}}^{(+)}\overbrace{TC_{RL}}^{(+)}\right)\right]\right\}}{|J|} \gtrless 0 \quad \text{(a.31)}$$

Effect of an Increase in Walking Distance, *WD*

An increase in walking distance causes the following change in transit demand

$$\frac{d\overline{TD}}{dWD} = \frac{\begin{vmatrix} 1 & -TC_{AS} & TC_{WD} & -TC_{RL} & 0 \\ -AS_{TC} & 1 & 0 & 0 & -AS_{D} \\ -TD_{TC} & -TD_{AS} & TD_{WD} & -TD_{RL} & 0 \\ -RL_{TC} & 0 & 0 & 1 & 0 \\ 0 & -D_{AS} & 0 & -D_{RL} & 1 \end{vmatrix}}{|J|} =$$

$$= \frac{RL_{TC}TC_{WD}TD_{AS} + TC_{WD}TD_{TC} + TD_{WD} - RL_{TC}TC_{RL}TD_{WD} + AS_{TC}(TC_{WD}TD_{AS} - TC_{AS}TD_{WD}) +}{|J|}$$

$$\frac{+AS_{D}\{D_{RL}RL_{TC}(TC_{WD}TD_{AS} - TC_{AS}TD_{WD}) - D_{AS}[TC_{WD}TD_{TC} + TD_{WD} + RL_{TC}(TC_{WD}TD_{RL} - TC_{RL}TD_{WD})]\}}{|J|} \qquad \text{(a.31)}$$

Effect of an Exogenous Change in Trip Chaining and Residential Location

This comparative-static is used to assess the magnitude of an exogenous change affecting both trip chaining and density. In particular, it is used to assess the extent of the impact of a change in distance to the nearest subcenter, an exogenous variable appearing on equation (4.13) and equation (4.17) of Model III. This change is measured as

$$\frac{d\overline{TD}}{dsubc\_dist} = \frac{\begin{vmatrix} 1 & -TC_{AS} & TC_{\varphi} & -TC_{RL} & 0 \\ -AS_{TC} & 1 & 0 & 0 & -AS_{D} \\ -TD_{TC} & -TD_{AS} & 0 & -TD_{RL} & 0 \\ -RL_{TC} & 0 & 0 & 1 & 0 \\ 0 & -D_{AS} & D_{\varphi} & -D_{RL} & 1 \end{vmatrix}}{|J|} =$$

$$= \frac{TC_{\varphi}(AS_{TC}TD_{AS} + RL_{TC}TD_{RL} + TD_{TC}) + AS_{\varphi}\{TC_{\varphi}[D_{RL}RL_{TC}TD_{AS} - D_{AS}(RL_{TC}TD_{RL} + TD_{TC})] +}{}$$

$$+D_{\varphi}[(1 - RL_{TC}TC_{RL})TD_{AS} + TC_{AS}(RL_{TC}TD_{RL} + TD_{TC})]\} \qquad \text{(a.32)}$$

Effect of an Exogenous Change in Activity Space

An exogenous change affecting the activity space has the following impact on the demand for transit

$$\frac{d\overline{TD}}{dAS_{\varphi}} = \frac{\begin{vmatrix} 1 & -TC_{AS} & 0 & -TC_{RL} & 0 \\ -AS_{TC} & 1 & AS_{\varphi} & 0 & -AS_{D} \\ -TD_{TC} & -TD_{AS} & 0 & -TD_{RL} & 0 \\ -RL_{TC} & 0 & 0 & 1 & 0 \\ 0 & -D_{AS} & 0 & -D_{RL} & 1 \end{vmatrix}}{|J|} = \frac{AS_{\varphi}[(1-RL_{TC}TC_{RL})TD_{AS}+TC_{AS}(RL_{TC}TD_{RL}+TD_{TC})]}{|J|} \quad \text{(a.33)}$$

This comparative-static is used to measure the change in transit demand due a change in retail establishment density (*r_estd*), an exogenous variable appearing in equation (4.14) in Model III.

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