Sampling-Based Approach for Incorporating Road Capacity Uncertainties into the Transportation Planning Process

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ABSTRACT

In the transportation planning literature, it is now well accepted that capturing uncertainty while evaluating transportation systems is important for arriving at better planning decisions. However, considering uncertainty presents numerous additional computational and theoretical challenges. This paper proposes an analytical methodology and efficient solution procedure that consider road capacity uncertainties as part of the overall transportation network model. The road capacity was considered as a random variable with certain distribution conditional to day-to-day roadway traffic conditions. For solution procedure, Sample-Average Approximation (SAA) was employed to generate plausible realizations of link capacity values from a multi-dimensional distribution and to solve the stochastic programming.

INTRODUCTION

There has been an increasing recognition of the need for effective approaches that incorporate the impact of uncertain events in transportation network models. It has long been recognized in the literature that capturing uncertainty in transportation system evaluation is important for arriving at better planning decisions (Mahmassani, 1984). However, planning by considering the effects of uncertain events, such as potential loss of vital transportation links as a result of incidents or accidents, adverse weather conditions and unfavorable road geometries such as steep climbs or lack of shoulders presents numerous theoretical and practical challenges.

This study is concerned with the development of transportation planning models considering risk of uncertain events at the network level. The road capacity is considered as a random variable with a certain distribution conditional to the day-to-day roadway traffic conditions. The distribution of highway capacities due to random events were determined through the identification of historical accident frequencies for our network links from corresponding real-world databases and roadway capacity reductions from the literature. We then extended the Traffic Assignment Model with Capacity Uncertainty (TAMCU) for investigating the general travel time reliability and the impact of network link risk. The solution procedure for TAMCU relies on an approach referred to as Sample-Average Approximation (SAA), which has been extensively used to solve the stochastic programming problems, such as asset investment problem (Blomvall and Shapiro, 2007) and structure reliability analysis (Tsompanakis, Y. et al., 2008).

PROPOSED METHODOLOGY AND APPROACH

The traffic assignment has found significant application since Wardrop (1952) under the standard assumption of deterministic origin-destination demand and deterministic link capacity conditions. Typically, two types of traffic assignments are performed: User Equilibrium (UE) - and System Optimum (SO). The formulation for each is given as follows (Sheffi, 1981):

$$
\begin{aligned}
&SO: \text{ Min} \sum_a x_a c_a(x_a) \text{ or UE : Min} \int_0^{x_a} c_a(\omega)d\omega \\
&s.t. \\
&\sum_k h_k^{rs} = q_{rs} \qquad \forall \text{r, s} \\
&h_k^{rs} \geq 0 \qquad \forall \text{r, s} \\
&\text{x}_\text{a} = \sum_\text{r}\sum_\text{s}\sum_\text{a} \text{h}_\text{k}^\text{rs}\delta_\text{a,k}^\text{rs} \ \forall a
\end{aligned}
\tag{1}
$$

Where q_{rs} is the OD flow from origin *r* to the destination *s*, h_k^{rs} is the flow on the path k from *r* to *s*, $\delta_{a,k}^{rs}$ is a binary value indicating that link *a* exists on path *k* between *r* and *s*, x_a is the flow on link *a*, and c_a is the cost of link *a*.

Traditionally, the adequacy of a road network is evaluated based on deterministic values of the network capacity and the required demand level. In fact, for certain time and day, link capacity is affected by various link states such as accident or incident, weather, geometric conditions et al. In general, the link states may represent subjective definitions by the planner of the successful function of a link (Bell & Iida, 1997), such as the probability distribution of each link state. The capacity for each link with different states can be represented by probability functions as follows:

$$C_{ir} = f(S_{i1}, S_{i1}, \ldots, S_{im}), i = 1,2,\ldots,m; \tag{2}$$

Where C_{ir} is the actual capacity of the link i; $S_{i1}, S_{i1}, \ldots, S_{im}$ are probability-based link states, which are assumed to be statistical independent among links. For example, if two link states, accident and weather condition, are considered and link capacity formula is assumed to be a linear function then the detailed probability-based link capacity can be represented as follow:

$$C_{ir} = (1-\alpha_{ia})(1-\alpha_{iw})C_i \tag{3}$$

Where C_i is the recommended value of link capacity, which can be found in *HCM 2000*. α_{ia} and α_{iw} are capacity reduction coefficients due to accident and weather states, respectively.

Note that the link capacity in (3) is for a certain time and day. In this study, we define link capacity distribution for certain time period as the combined capacity values given by equation (3) for different days. For example, suppose that for a certain time period (e.g. PM peak period), the link capacity can be calculated for several days (e.g. one year) with corresponding link states. Thus, results of capacity values are considered as link capacity distribution for PM peak period in one year.

Using the above link capacity distribution definition, suppose we have *m* links in the network, we view the uncertain capacity parameter vector $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$ as a variable vector and for each item ξ_i it has a probability distribution p_i. We then formulate the following stochastic programming problem:

$$Min\{f(\text{x}) = \text{E}[F(\text{x}, \xi(\omega))]\} \tag{4}$$

Where $F(x, \xi)$ is either SO or UE formulation of traffic assignment problem and $\xi(\omega)$ is the link capacity which is a random vector having certain probability distribution calculated by equations (2) and (3). The objective is to obtain the expected value of either SO or UE assignments considering all of the potential realistic link capacity combinations. The above formulation can also be considered as a bi-level programming with upper level shown in equation (4) and lower level shown in equation (1).

Sample-Average Approximations (SAA)

For solving the above stochastic programming problem shown in equation (4), a common approach is to replace the probability distribution of $\xi(\omega)$ by a finite supported measure; that is, $\xi(\omega)$ has a finite number of possible realizations, called scenarios $(\xi_1, \xi_2, \ldots, \xi_k)$ with respective probabilities $p_k \in (0,1), k = 1,2,\ldots,K$. For such problems, the expected value function in equation (4) can be written as the following finite sum

$$Min\{\hat{f}_k(x) = \sum_{k=1}^{K} p_k F(x, \xi_k)\} \tag{5}$$

Suppose that we can generate the above scenarios with the same probability, then (5) can be written as:

$$Min\{\hat{f}_k(x) = K^{-1} \sum_{k=1}^{K} F(x, \xi_k)\} \tag{6}$$

Where $F(x,\xi)$ is the optimal value of either SO or UE assignment and ξ_k denotes the link capacity vector. K is the sampling size. For any fixed $x \in X$, we obtain $\hat{f}_k(x)$ as an unbiased estimator of the expectation $f(\mathrm{x})$, and by the Law of Large Number that $\hat{f}_k(x)$ converges to $f(\mathrm{x})$ when $K \to \propto$. In scenario-based algorithms, each scenario can often be considered independently from other scenarios. This makes such algorithms well suited for implementation on parallel computing platforms, where different processors execute the independent portions of the computation in parallel.

However, computations required by (5) or (6) are not practical to execute when the random data vector ξ has large number of components with possible values. For example, if vector ξ has 200 components, which means 200 links in the network, and suppose that each component has three possible values, then the total number of scenarios is 3^{200}. It is of course not possible to repeatedly solve the problem in (1) for all combinations of possible capacity realizations.

We can however use sample-average approximation via sampling techniques by randomly selecting subsets of the set $(\xi_1, \xi_2, \ldots, \xi_k)$ to obtain approximate solutions. This approximate objective function, known as a sample-average approximation (SAA) of $f(\mathrm{x})$, is then minimized using a deterministic optimization algorithm. In this study, we employed a sampling approach, in which a sample is selected from $(\xi_1, \xi_2, \ldots, \xi_k)$ and corresponding approximation to $F(x)$ is defined from this sample. The detailed solution procedure for SAA adopted in this study is shown in FIGURE 1. The reduction factor includes different disadvantages for road capacity, such as extreme weather condition, incident, accident and etc.

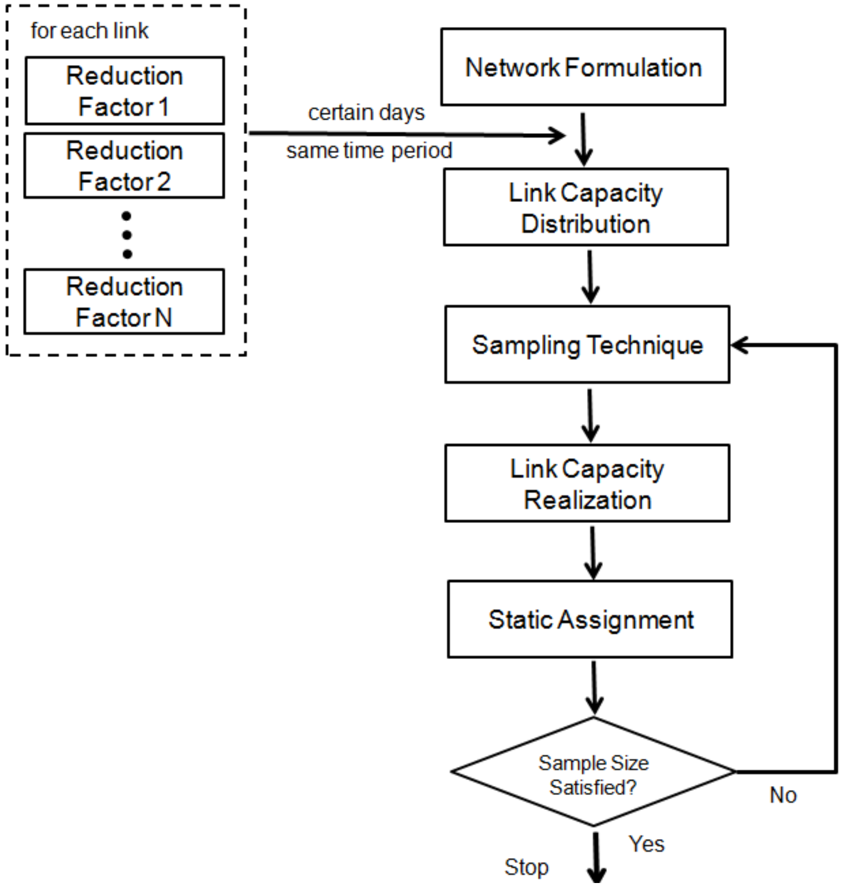

FIGURE 1 Representation of SAA Solution Steps

CASE STUDY

We apply the proposed methodology to a case study of no-notice evacuation planning study. In this paper, we first investigate the impact of capacity uncertainty on average evacuation times under different demand scenarios. The main insight from the analysis of our results is that when the evacuation demand increases, the impact of capacity uncertainties becomes more important. The difference in stochastic and deterministic capacity scenarios increased from 8.11% to 11.36% in average.

In addition to average evacuation times, variation of evacuation times and their distribution under stochastic capacities are studied. For instance, from distributional properties, the percentile of the evacuated population vs. time graphs can be obtained. This kind of probabilistic analysis approach provides the decision maker with better insights to the problem at hand. Which can estimate the probability of having a certain number of people in danger after the passage of certain amount of time. In FIGURE 2 the percentage of evacuated populations under different demand conditions are compared. For high demand scenario, it takes around 10 minutes to evacuate 50% of the demand considering both full and stochastic capacity scenarios. However, the average evacuation times to fully clear the impact area increase from 12 minutes to 16 minutes for full and stochastic capacity scenarios, respectively. Thus, capacity uncertainties increase the evacuation time by 25%.

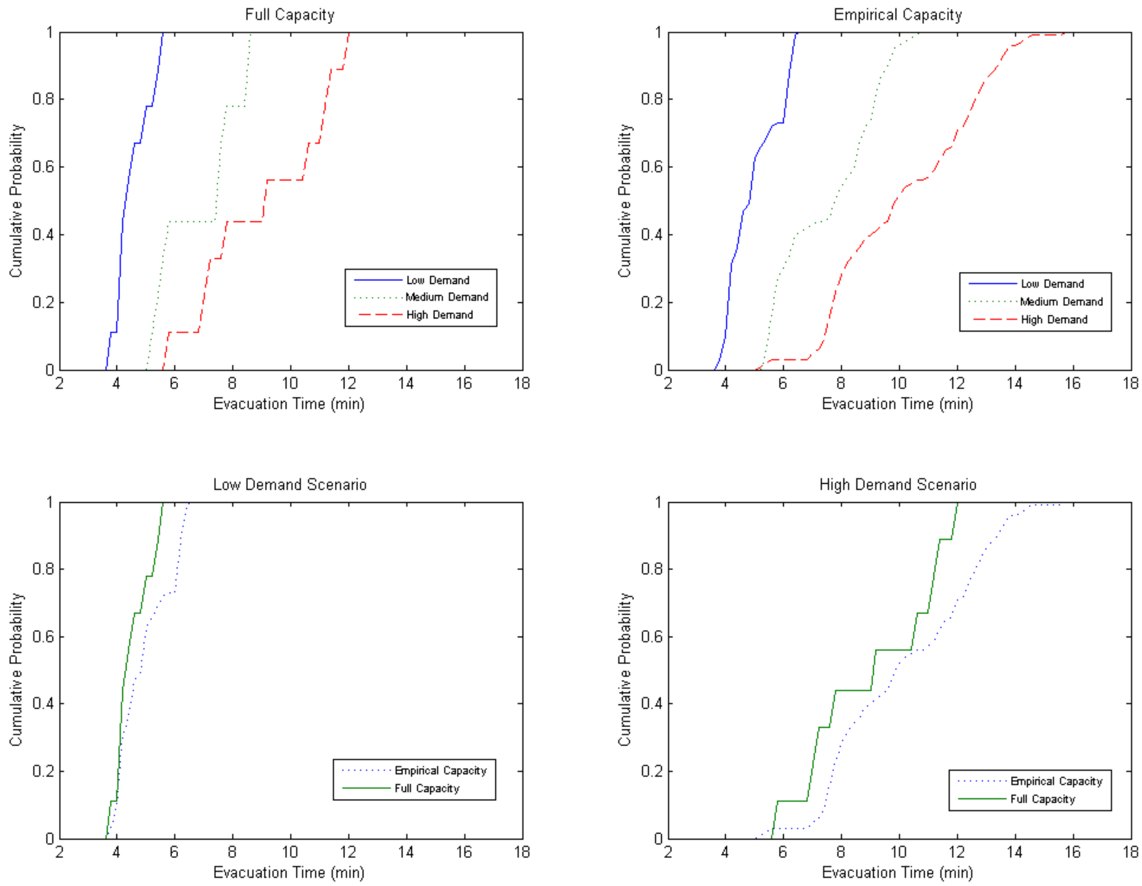

FIGURE 2 Percentage of Evacuated Population under Different Demand

CONCLUSION AND FUTURE RESEARCH

In this study, an analytical methodology and efficient solution procedure are proposed for the development of transportation plans considering road capacity uncertainty due to the changes in day-to-day roadway traffic conditions. SAA methodology is then employed for capturing network capacity uncertainty and solving the resulting stochastic programming problem. The presented methodology differs from earlier work on uncertainty analysis in transportation planning because it employs stochastic programming formulation to capture the impact of real-world conditions on link capacities rather than using scenario analysis or simple assumptions. This methodology can be potentially very useful for improving transportation plans by capturing realistic conditions.

Our methodology can be tested for very large regional transportation networks by considering a number of efficient sampling techniques. It will thus be beneficial if other sampling algorithms are used to estimate the upper or lower bounds of the proposed bi-level stochastic programming approach. However, it should be noted that the proposed methodology and solution procedure are highly dependent on the availability of empirical data. In this study we used an empirical accident database to estimate potential risks. More detailed analysis of similar databases for better estimation of link risk functions can be an important improvement in the future.

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