Accounting for Spatial Dependency in Joint Models of Motorized and Non-motorized Travel

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INTRODUCTION

Increased auto dependence has raised many societal concerns such as physical inactivity, air pollution, climate change, and increased traffic congestion. In order to address these concerns, policy makers around the world are seeking strategies that would encourage people to drive less and walk/bike more (U.S. Department of Health and Human Services, 1996; Sallis et al, 2004). In particular, many are interested in identifying the most promising built environment (BE) design elements to promote active travel. This policy interest raises two requirements on travel modeling.

First, given that a policy variable may have substitutive, complementary and synergistic effects between different modes of transportation (Guo et al, 2007), the travel outcome of the policy variable on various modes need to be modeled simultaneously. This has been accomplished in only a few past analyses of trip frequencies. For example, Guo et al (2007) adopted a bivariate ordered probit model structure to account for intra-personal correlations between motorized and non-motorized trip frequencies. The bivariate structure was shown to outperform a pair of independent ordered probit models. In Cao et al. (2006), trip frequencies of automobile, transit and walking were regressed against BE and other factors using a seemingly unrelated regression (SUR) model to account for correlations among the mode-specific linear regression. No test statistic was provided in the study to compare the model’s goodness-of-fit against independent regressions.

The second modeling requirement arises from the recognition that people travel in shared space. As stated in Tobler’s first law of geography: “Everything is related to everything else, but near things are more related than distant things.” (Tobler, 1970, p.236). Thus, individuals located closer to each other are likely to share similarly in their travel environment (this could be the physical or the social environment). Failure to explicitly account for this spatial dependency, or spatial correlation, could lead to biased estimation of policy impact on travel behavior. To the best of the authors’ knowledge, no past studies have examined this issue of spatial dependency in the context of joint motorized and non-motorized travel modeling.
This paper aims to enhance public investment decision-makers’ ability in identifying effective walking/biking promotive strategies by developing an econometric model capable of meeting the abovementioned requirements. Specifically, the Spatial Seemingly Unrelated Regression (SSUR) model first proposed by Anselin (1988) is adopted to jointly model individuals’ daily Vehicle Miles Traveled (VMT) and Miles Walked/Biked (MWB) as a function of BE factors while controlling for a wide range of additional variables. Here, VMT and MWB are chosen as measures of travel outcome because transportation investment project costs (such as construction and operational costs) and benefits (such as crash reduction, emission reduction, calories burnt) are commonly measured in terms of miles traveled. Choosing VMT and MWB as the model outcome variable would allow policy makers to assess the return on their investment in a relatively straightforward manner.

**MODEL STRUCTURE**

The SSUR model – first proposed by Anselin (1988) – is a spatial analogue of serial autocorrelation in the intra-equation disturbances of a time series model. In this study, the travel model is specified to include the following two equations for each observed individual \( n, n = 1 \ldots N \):

\[
\begin{align*}
f_{1n} &= X_n \beta_n + e_n \\
f_{2n} &= Y_n \alpha_n + v_n
\end{align*}
\]

where \( f_{1n} \) is the daily MWB by person \( n \); \( f_{2n} \) is daily VMT by person \( n \); \( X_n \) and \( Y_n \) are vectors of regressors that describe the traveler, neighborhood, and travel day characteristics; \( \beta_n \) and \( \alpha_n \) are the corresponding vectors of model parameters to be estimated; \( e_n \) and \( v_n \) are the disturbance terms. This system of equations can be expressed by a stacked model:

\[
\begin{bmatrix}
f_1 \\
f_2
\end{bmatrix} =
\begin{bmatrix}
X & 0 \\
0 & Y
\end{bmatrix}
\begin{bmatrix}
\beta \\
\alpha
\end{bmatrix}
+ 
\begin{bmatrix}
e \\
v
\end{bmatrix}
\]

where
\[ e = \lambda_1 W e + \mu_1 \]
\[ v = \lambda_2 W v + \mu_2 \]
or more parsimoniously as:
\[ h = ZA + \varepsilon \]
\[ \varepsilon = \lambda W \varepsilon + \mu \]
Here, \( \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \); \( \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \); \( \varepsilon = \begin{bmatrix} e \\ v \end{bmatrix} \); and \( W \) is the spatial weight matrix.

The error terms in the above system of equations follow a spatial autoregressive process within each equation (with a different autocorrelation coefficients \( \lambda_1, \lambda_2 \)), as well as being correlated between equations. The spatially dependent error vector \( \varepsilon \) can be considered as a transformation of the independent \( \mu \), as:
\[ \varepsilon = (I - \lambda W)^{-1} \mu \]

In the above specification of SSUR model, \( \mu \) would be assumed to satisfy the following conditions:
\[ E[\mu] = 0 \], and
\[ E[\mu_1, \mu_2'] = \sigma_{12} \circ I \],

It follows that the error covariance matrix \( \Omega \) takes the following form:
\[ \Omega = E[\varepsilon, \varepsilon'] = B(\Sigma \otimes I)B' \]
where
\[ B = \begin{bmatrix} (I - \lambda_1 W)^{-1} & 0 \\ 0 & (I - \lambda_2 W)^{-1} \end{bmatrix} \]

Since \( W \) is constant across observations, \( B \) can also be expressed as
\[ B = [I - (A \otimes W)]^{-1} \]
where \( A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \) and \( I \) is a 2*N by 2*N identity matrix.
The SSUR model was chosen over the SUR model (consisting of a system of two independent linear regression models) for two reasons. First, the SSUR structure allows the residual terms in the two equations to correlate with each other. This accounts for any unobserved correlation between an individual’s MWB and VMT on a given day. Such correlation could arise from, for example, personal attitudes towards environmental conservation that our survey data was unable to capture. Second, the SSUR structure accounts for the possible spatial correlation across observations about individual travelers. Failure to account for such inter-equation and inter-observation correlation effects could lead to erroneous estimation results (Anselin, 1988). The similarities and differences between a SSURE model and a SUR model are summarized in Table 1 below.

Table 1. SUR vs. SSUR model structure

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>SUR model</th>
<th>SSUR model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stacked model</td>
<td>[ f_1 = X 0 \beta + e ]</td>
<td>[ f_1 = X 0 \beta + e ]</td>
</tr>
<tr>
<td></td>
<td>[ f_2 = 0 Y \alpha + v ]</td>
<td>[ f_2 = 0 Y \alpha + v ]</td>
</tr>
<tr>
<td>Spatial error dependence</td>
<td>None</td>
<td>[ e = \lambda_1 W e + \mu_1 ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ v = \lambda_2 W v + \mu_2 ]</td>
</tr>
<tr>
<td>Parsimonious definition</td>
<td>[ h = Z \alpha + \varepsilon ]</td>
<td>[ h = Z \alpha + \varepsilon ]</td>
</tr>
<tr>
<td></td>
<td>[ \varepsilon = \lambda W \varepsilon + \mu ]</td>
<td></td>
</tr>
<tr>
<td>Error-covariance matrix</td>
<td>[ E[\varepsilon] = 0 ]</td>
<td>[ E[\mu] = 0 ] and [ E[\mu_1, \mu_2] = \sigma_{12} \circ I ]</td>
</tr>
<tr>
<td></td>
<td>[ E[\varepsilon, v'] = \sigma_{12} \circ ]</td>
<td>[ E[\varepsilon, v'] = \sigma_{12} \circ [(I - \lambda_1 W)(I - \lambda_2 W)]^{-1} ]</td>
</tr>
<tr>
<td></td>
<td>and,</td>
<td>and,</td>
</tr>
<tr>
<td></td>
<td>[ E[\varepsilon, \varepsilon'] = \Omega = \Sigma \otimes I ]</td>
<td>[ E[\varepsilon, \varepsilon'] = \Omega = B(\Sigma \otimes I)B' ]</td>
</tr>
</tbody>
</table>

DATA

Dane county, located in the southeastern part of Wisconsin, was chosen as the study area for this paper. Our primary source of travel data was the Wisconsin add-on sample of the 2001 National Household Travel Survey (NHTS), which contained all the trips made by members of each sampled household on a single day. Each household’s residential location as well as the trip origins and destinations of each recorded trip were geo-coded,
allowing for a high resolution spatial analysis. Distances of observed trips were estimated based on the shortest paths on the network. For each automobile trip, the distance traveled was divided by the occupancy rate to give the equivalent person vehicular miles traveled (PVMT). The PVMT were subsequently summed up for each individual to give the daily VMT. Similarly, the distances of all biking and walking trips made by the same individual were summed up to give that individual’s daily MWB. The VMT and MWB formed the dependent variables for subsequent analyses.

In addition to the 2001 NHTS data, a number of other data sources were used to derive three groups of environmental variables: (a) neighborhood measures, (b) regional accessibility measures, and (c) weather measures. The neighborhood measures were computed by first constructing 1/4-mile and 1-mile network buffers around each sampled household to represent two alternative ‘neighborhood’ definitions of the households (see Guo and Bhat, 2007, for a discussion on operational definitions of neighborhoods). The census, land use, and transportation network data were then overlaid onto these network buffers to obtain neighborhood-level measures of social-demographic distribution, land use coverage by type, land use mix, and transportation network characteristics. The inclusion of regional accessibility measures was motivated by our belief that an individual’s travel amount and mode preference depend not only on the environment surrounding his/her residence, but also how the residence relates spatially to the rest of the urban area. To compute the weather measures, each residence in the sample was linked to the closest NCDC precipitation and climate station using Euclidean distance measures. Maximum and minimum temperatures, as well as total daily precipitation, were extracted for each individual’s survey day from his/her corresponding precipitation and climate station. A set of dummy variables were also derived to indicate whether the surveyed date fell on the weekend and which season to account for any temporal variations in travel.

Our final sample included 2487 persons, each associated with the two dependent variables and a rich array of person, household, and environmental measures as explanatory variables for SSUR model estimation. Also need for model estimation is the weight matrix, $W$. In this particular empirical application, $W$ is defined using the
inverse Euclidean distance between sampled households. For each household, the inverse
distance to the other household is taken as weight provided if they are within a distance
of two miles.

MODEL ESTIMATION

The above described data were used to estimate a SUR model and a SSUR model. The
generalized least squares (GLS) estimator for the unknown parameters in the
conventional SUR model is given by:

$$\hat{\Lambda} = [Z'\Omega^{-1}Z]^{-1}Z'\Omega^{-1}h = [Z'(\Sigma^{-1} \otimes I)Z]^{-1}Z'(\Sigma^{-1} \otimes I)h.$$ 

The ML estimation of the SSUR model entails an iterative approach to solve for $\Sigma$, $\lambda$
and parameter estimates as outlined by Anselin (1988) and described below.

Step 1. Estimate each equation using the ordinary least squares method to obtain the
initial set of equation specific residuals $e$ and $v$.

Step 2. Given $e$ and $v$, estimate $\lambda$ by optimizing the concentrated log likelihood
defined below:

$$L_c = C - (N/2)\ln[(1/N)e'(I - \lambda_iW)'(I - \lambda_iW)e] + \ln[I - \lambda_iW]$$

Step 3. Given $\lambda$, compute the estimate of $\Sigma$ using the following expression:

$$\Sigma = (1/N)\begin{bmatrix} e'(I - \lambda_1W)'(I - \lambda_iW)e & e'(I - \lambda_1W)'(I - \lambda_2W)e \\ v'(I - \lambda_2W)'(I - \lambda_iW)e & v'(I - \lambda_2W)'(I - \lambda_2W)e \end{bmatrix}$$

Step 4. Use the $\Sigma$ from the previous step as the starting value in estimating a
standard SUR model on spatially transformed $h$ and $z$ given by:

$h^* = (I - \lambda W)h$, and

$z^* = (I - \lambda W)z$.

The GLS estimates at this step for the spatially transformed variables are
given by:

$\Delta = [Z'\Omega^{-1}Z]^{-1}Z'\Omega^{-1}h$, where

$$Z'\Omega^{-1}Z = \begin{bmatrix} \sigma^{11}X'(I - \lambda_iW)'(I - \lambda_iW)X & \sigma^{12}X'(I - \lambda_iW)'(I - \lambda_2W)Y \\ \sigma^{21}Y'(I - \lambda_2W)'(I - \lambda_iW)X & \sigma^{22}Y'(I - \lambda_2W)'(I - \lambda_2W)Y \end{bmatrix},$$

and
\[
Z'\Omega^{-1}h = \begin{bmatrix}
\sigma^{11}X'(I - \lambda_1 W)'(I - \lambda_1 W)f_1 + \sigma^{12}X'(I - \lambda_2 W)'(I - \lambda_2 W)f_2 \\
\sigma^{21}Y'(I - \lambda_2 W)'(I - \lambda_2 W)f_1 + \sigma^{22}Y'(I - \lambda_2 W)'(I - \lambda_2 W)f_2
\end{bmatrix}
\]

Step 5. If the parameter estimates did not converge, new residuals are obtained using the current parameter estimates from step 4 and \(\lambda\) is solved from the following simultaneous non-linear equations:

\[
\begin{align*}
tr[W(I - \lambda_1 W)^{-1}] &= \sigma^{11}e'W'(I - \lambda_1 W)e + \sigma^{12}e'W'(I - \lambda_2 W)v \\
tr[W(I - \lambda_2 W)^{-1}] &= \sigma^{21}v'W'(I - \lambda_1 W)e + \sigma^{22}v'W'(I - \lambda_2 W)v
\end{align*}
\]

Step 6. With these new values of \(\lambda\), new estimate for \(\Sigma\) is derived using the same expression as in step 3 but substituting the OLS residuals with the current SUR residuals.

Step 7. Go back to step 4 and continue the iterations until the convergence criterion is met in step 5.

RESULTS

The estimation results of both the SUR and SSUR models are reported in Table 2. Due to word limit considerations, parameter estimates for the exogenous variables are not individually discussed here. Rather, the discussion below focuses on the comparison between the two models.

First of all, the estimation results do not suggest any dramatic change in the magnitude of the various parameter estimates. However, many of the person characteristics and some BE variables appear to have lower statistical significance in the SSUR model than in the SUR. The two BE variables (retail accessibility and length of no sidewalk) that appear to be the most effective biking/walking-promotive in SUR model are also statistically significant in the SSUR model, suggesting their potential in simultaneously promoting biking and walking along with decreasing driving.

With regard to the overall fit of the model, SSUR has a higher system r-squared (0.15 vs. 0.12). This better fit of SSUR model can be explained on account of addressing the significant spatial autocorrelation effect in the sample. Teasing out spatial autocorrelation may have also led to the reduced cross-equation correlation between the error terms. As
shown in Table 3, there is a significant change in the variance matrix ($\Sigma$) and slight decrease of the cross equation correlation between SUR and SSUR model.
### Table 2. SUR and SSUR model estimation results

| Explanatory Variables | SUR Model | | | Spatial SUR Model | | |
|-----------------------|-----------|-----------|-----------------|-----------|-----------|
|                       | MWB       | VMT       | MWB             | VMT       |
|                       | Coef.     | z-stat    | Coef.           | z-stat    |
| Person/Household/Trip Day Characteristics | | | | | |
| Person is employed    | 0.1663    | 2.976***  | 14.811 9.583*** | 0.0610    | 0.894     | 16.9346 7.994*** |
| Person is young (17 to 30 years old) | 0.2255    | 2.929***  | -- --           | 0.1271    | 1.3674    |
| Person is Caucasian   | 0.2729    | 2.761***  | -- --           | 0.2582    | 2.172**   |
| Person holds a driving license | -- --     | 11.6439 12.446*** | | 10.5879 8.136*** |
| Person has a degree (Bachelor’s or higher) | -- --     | 2.3258 3.570*** | | 2.0657 2.281*** |
| Number of bicycles owned by household | 0.1480    | 8.309***  | -- --           | 0.1452    | 6.524***  |
| Household has no car  | 0.3548    | 1.803*    | -- --           | 0.0439    | 0.186     |
| Family income per year (in $10,000) | -- --     | 0.2956 2.266** | | -0.1229 0.661 |
| Number of cell phones in household | -- --     | 0.8638 2.806*** | | 1.5234 3.480*** |
| Housing type is either an apartment or a dormitory | 0.1704    | 1.985**   | 2.2296 2.495**  | 0.1968    | 1.800*    | 21.285 1.578 |
| Lowest temperature on travel day | 0.0073    | 4.805***  | -- --           | 0.0066    | 3.609***  |
| Travel day is on a weekend | -- --     | -6.8482 -2.343** | | -13.1987 3.397*** |
| Built Environment Characteristics | | | | | |
| Regional factors      | | | | | |
| Rural setting         | -- --     | 1.3241 1.553 | | 0.9449    | 0.721     |
| Retail accessibility  | 0.0399    | 3.341***  | -0.5785 -3.438*** | 0.0437    | 2.693***  | -0.0145 0.053 |
|                     | -- --     | -1.2072 -5.624*** | | -1.7220 5.601*** |
| Neighborhood socio-demographic composition | | | | | |
| % high income households in neighborhood – 1 mile buffer | -0.9233    | -3.846***  | 9.7954 3.561*** | -0.8449    | -2.767***  | 15.9405 3.782*** |
| Household density (per acre) – ¼ mile buffer | -- --     | 0.2823 2.833*** | | 0.2084    | 1.167     |
| Neighborhood land use characteristics | | | | | |
| Land use mix – 1 mile buffer | -0.5786    | -3.466***  | -0.0547 -2.874*** | -0.3574    | -1.684*   | -10.0319 3.207*** |
|                     | -- --     | 4.7199 4.334*** | | 4.5087    | 2.889***  |
|                     | -- --     | 8.1199 1.786* | | 17.1592 2.818*** |
| Neighborhood transportation network characteristics | | | | | |
| Length of roadway with no sidewalk – 1 mile buffer | -0.0483    | -3.288***  | 0.3397 2.128**  | -0.0554    | -2.784***  | 0.6447 2.390** |
| Length of roadway with bike lane – ¼ mile buffer | 0.2140    | 2.265**   | -- --           | 0.1005    | 0.801     |
| Number of intersections (per acre) – ¼ mile buffer | 0.0503    | 2.261**   | -- --           | 0.0160    | 0.550     |
| r-squared            | 0.0511    | 0.1898    | 0.0429          | 0.236     |
| system r-square      | 0.1261    | 0.1507    | 0.0006          | 0.0015    |
| autocorrelation coefficient | | | | | |

*** indicates significance at 99% level, ** indicates 95% level, * indicates 90% level, All other variables are not significant at 90% level.
Table 3. Variance, Cross equation correlation between SUR and SSUR

<table>
<thead>
<tr>
<th></th>
<th>SUR</th>
<th>SSUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MWB</td>
<td>2.6744</td>
<td>2.7389</td>
</tr>
<tr>
<td>VMT</td>
<td>-2.3153</td>
<td>-2.5286</td>
</tr>
</tbody>
</table>

Cross-equation correlations

<table>
<thead>
<tr>
<th></th>
<th>SUR</th>
<th>SSUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MWB</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>VMT</td>
<td>-0.0694</td>
<td>-0.0737</td>
</tr>
</tbody>
</table>

CONCLUSIONS

This paper has examined the application of the SSUR model to account for the inter-equation correlation due to unobserved personal characteristics and the spatial error autocorrelation among the error terms of each equation. The estimation results suggest significant presence of spatial dependency and a marginally improved goodness-of-fit of the SSUR model over the conventional SUR model, at least in this particular empirical context.

The comparison between the two sets of parameter estimates suggests no dramatic change in the signs of parameters between the two models. Of note are the two BE-related parameters that consistently have opposite signs in the MWB and VMT equations in both models. The opposite signs suggest that both strategies – increased retail accessibility and improved prevalence of sidewalks within 1 mile neighborhood buffers – have the substitutive effect of simultaneously increasing an individual’s level of walking/biking and reducing distance traveled by vehicular modes. This finding, in part, concurs with the theory of Neo-urbanism.

While most parameter estimates are similar in magnitude between the two models, the corresponding estimates are quite different for some variables (for example, travel day being on the weekend, retail accessibility, and land use mix). If these parameter estimates are subsequently used in, for example, assessment of return on alternative BE investment
(see Guo and Gandavarapu, 2009), these differences in estimates may lead to very different policy recommendations. This points to the importance of selecting the ‘right’ model structure for such policy assessment.

Given that this is one of the first studies to apply the SSUR model in travel modeling, future empirical applications are needed to further assess the promise of such a model. One of the lessons learnt in this study is that currently none of the public or proprietary econometric modeling software supports the estimation of SSUR models. Making the model estimation procedure available to the research community and policy analysts is key to promoting the consideration and application of such advanced spatial econometric models in travel modeling.

REFERENCES


