A Gap Function-Based Static Traffic Assignment Model with Multiple User Classes under Stochastic Capacity

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Abstract

Conventional static traffic assignment methods generally assume users' perfect information and deterministic capacity. To capture day-dependent travel times under stochastic (day-varying) capacities, and evaluate the potential benefits of various information provision strategies, this paper develops a new class of static traffic assignment methods. This proposed method categorizes commuters into two classes; (1) users with perfect traffic information every day, and (2) users with imperfect information derived from average network conditions. A user equilibrium model is formulated based on a gap function approach to describe the route choices of these two user classes under stochastic capacities. Two types of system optimum models, namely, steady-state and day-dependent models are also formulated under the same modeling framework. A numerical example is presented to demonstrate how the proposed model can be used to rapidly quantify the value of information.

Keywords: stochastic capacity; gap function; static traffic assignment; system-optimum; value of information

1. Introduction

Conventional static traffic assignment approaches are generally developed based on users' perfect knowledge of network and deterministic capacity assumptions. However, recent empirical research (Brilon, et al. 2005, 2007) indicates that highway capacity reveals all natures of random variables. Even for constant geometric, traffic, environmental, and operational conditions, capacities vary with time over a certain range around the mean values. More precisely, highway capacity is the result of driver behavioral interaction, and also depends on many external factors such as accidents, incidents, weather, or work zones. In addition, not all network users can have access to "perfect network information". Motivated by the practical need for a method that can capture (a) day-varying travel times and (b) potential benefits of information provision strategies, this paper proposes a new type of static traffic assignment method, which considers different travel time perception models under stochastic capacity.

2. Effect of stochastic capacity on network conditions

To elaborate the effects of stochastic capacity across different days on travel time perception and route choices, Fig.1 gives a simple illustrative example. Given two alternative paths between an OD pair, suppose the primary path capacity varies on different days, and the alternative path has a constant capacity. A BPR function with $\alpha = 1.5$ and $\beta = 4$ is used for each of the paths; while the total OD demand is given a value of 8000 veh/h during the analysis period each day.

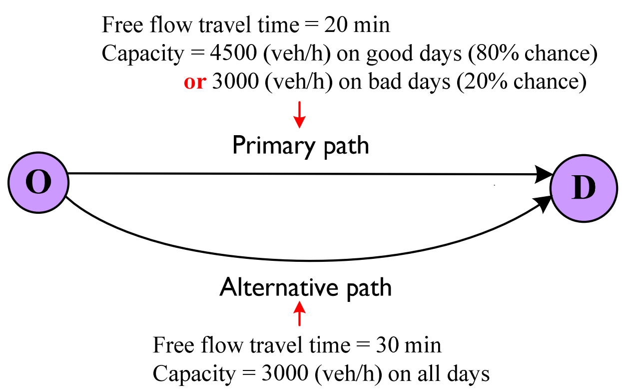

Fig. 1. Network used in the illustrative example

Shown as Fig.2, the curves represent the flow-travel time relationship corresponding to expected performances as well as actual performances on good and bad days. A similar analysis focused on good and bad day fluctuations can be found in the research by de Palma and Picard, 2005. This paper extends their approach to construct more rigorous optimization formulations and computationally efficient solution algorithms. In Fig.2, point A represents an expected user equilibrium conditions achieved when users only have perceive average travel time conditions through long-term experiences., Under this "expected value" solution, the path flow proportions are the same across different days, shown as the vertical dashed line. On the primal path, the resulting travel times can be further estimated using the BPR function. That is, the travel time corresponds to D on bad days or E on good days, and point A represents the average travel time. On the alternative path, as its capacity is constant, the travel time is constant as point A.

That is, the average travel times across multiple days on different routes (with flow) are the same in this "expected value" UE conditions with stochastic road capacity. On the other hand, the route travel times on each day are different and can be viewed as "disequilibrium" for an observer with only data from a single day, as the non-shortest path of each day still carry positive flow volume.

If assuming all the network users are provided with perfect network and traffic knowledge every day, the network flow conditions lead to point B on those bad days or point C on the good days. This example demonstrates different possible travel time perception and responses of various user information classes to stochastic capacity.

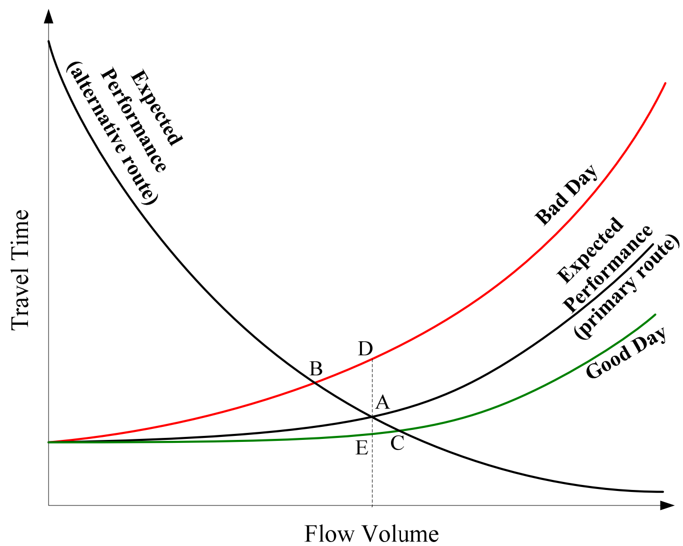

Fig. 2. Illustration for impacts of stochastic capacities and information provision on route choice

Corresponding to Fig.2, the coordinates (i.e. values) of the traffic flow volume and travel time conditions from A to E are detialed in Tables 1 and 2.

Table. 1. Estimated flows and travel times on primary path

	Calculated travel time on primary path (min)	Calculated flow on primary path (veh/h)
A	32.2	5503
B	37.1	4636
C	30.6	6172
D	56.4	5503
E	27.1	5503

Table. 2. Estimated flows and travel times with real-time traffic information

		Day 1	Day 2	Day 3	Day 4	Day 5
	Capacity(veh/h)	3000	4500	4500	4500	4500
Primary path	Travel Time (min)	B: 37.1	C: 30.6	C: 30.6	C: 30.6	C: 30.6
Alternative path	Travel Time (min)	B: 37.1	C: 30.6	C: 30.6	C: 30.6	C: 30.6

Using this framework, the benefits of information provision can be clearly quantified. For example, if all the users in the networks are provided with perfect information, then on bad days, the average travel time is 37.1 min, compared to the expected value solution, which can be calculated as (56.4×5503+32.2×(8000-5503))/8000=48.4 min.

Table 3. Day-dependent travel times without real-time traffic information

		Day 1	Day 2	Day 3	Day 4	Day 5	Avg.
Primary path	Capacity (veh/h)	3000	4500	4500	4500	4500	
	Travel Time (min)	E: 56.4	D: 27.1	D: 27.1	D: 27.1	D: 27.1	32.2
Alt. path	Capacity (veh/h)	3000	3000	3000	3000	3000	
	Travel Time (min)	A: 32.2	A: 32.2	A: 32.2	A: 32.2	A: 32.2	32.2

3. Gap function-based static flow assignment with multiple information classes

3.1. Notations

The sets and subscripts, parameters and decision variables in the proposed flow assignment model are introduced as follows:

Indices:

i = index of origins, i = 1, …, I, where I is the number of origins

j = index of destinations, j = 1, …, J, where J is the number of destinations

p = index of paths, p =1, …, P, where P is the number of paths between an OD pair i and j

a = index of links, a =1, …, A, where A is the number of links in networks

d = index of days, d =1, …, D, where D is the number of days over analysis horizon

k = index for user classes. k =1 represents users with perfect knowledge of the capacity fluctuation from day to day; while k =2 represents users who make their route choice based on their expectations.

Input Parameters:

$c_{a,d}$ = capacity of link a on day d

$\overline{c}_a$ = expected capacity of link a over all the days over the time horizon

$q_d^{i,j}$ = OD demand between an OD pair i and j on day d

$\delta_{p,a}$ = link-path incidence coefficient, taking a value of 1, if link a is on path p, and 0 otherwise

α = market penetration rate of the perfect information users to the total OD demand

Decision variables:

$f_{k,p,d}^{i,j}$ = path flow of user class k on path p between an OD pair i and j on day d

$f_{p,d}^{i,j}$ = total path flow of path p between an OD pair i and j on day d

$C_{p,d}^{i,j}$ = generalized path cost between an OD pair i and j on day d

$\overline{C}_{p,d}^{i,j}$ = expected path cost between an OD pair i and j over the multi-day horizon

$v_{k,a,d}$ = link flow of user class k on link a on day d

$v_{a,d}$ = total link flow on link a on day d, $v_{a,d} = \sum v_{k,a,d}$

$C_{a,d}$ = generalized link cost on link a on day d, which is a function of capacity

$c_{a,d}$ and link flow $v_{a,d}$

$\pi_d^{i,j}$ = day-dependent minimum possible total generalized cost between i and j on day d

$\overline{\pi}^{i,j}$ = expected minimum possible total generalized cost between an i and j over the multi-day horizon

3.2. Model formulation

The proposed model incorporates the two user classes into a static traffic assignment framework under stochastic capacity, which varies on a daily basis. The objective function aims to minimize the gap between estimated path flows and user equilibrium conditions, where the first summation is for the user class with perfect traffic information, and the second summation for the users with imperfect information based on expected values.

Objective function:

$$\min Gap = \sum_d \sum_i \sum_j \sum_p \left[f_{1,p,d}^{i,j} \times \left(C_{p,d}^{i,j} - \pi_d^{i,j} \right) \right] + \sum_i \sum_j \sum_p \left[f_{2,p,d}^{i,j} \times \left(\overline{C}_{p,d}^{i,j} - \overline{\pi}^{i,j} \right) \right] \quad (1)$$

Constraints (2) and (3) show the relationship between OD demand and path flows for each class. As we have analyzed, the users with imperfect information are assumed to keep their route choices based on expected values. Then, they have day-invariant (or day-independent) route choices as Eq. (4). Eq. (5) builds up the connection between path flows and link flows consisting of two user classes; while Eq. (6) incorporates day-varying link capacity as an input parameter, and expresses generalized path costs as a function of link flows and capacity.

Constraints:

OD demand - path flow relationship for users with perfect information

$$\alpha \times q_d^{i,j} = \sum_p f_{1,p,d}^{i,j} \qquad \forall i, j, d \quad (2)$$

OD demand - path flow relationship for users with imperfect information (based on expected values)

$$(1-\alpha) \times \sum_d q_d^{i,j} = D \times \sum_p f_{2,p,d}^{i,j} \qquad \forall i, j, d \quad (3)$$

Day-invariant path flow for users with imperfect information

$$f_{2,p,d=1}^{i,j} = f_{2,p,d=2}^{i,j} = \cdots = f_{2,p,d=D}^{i,j} \quad (4)$$

Path flow - link flow relationship

$$v_{a,d} = \sum_i \sum_j \sum_p f_{1,p,d}^{i,j} \cdot \delta_{p,a} + \sum_i \sum_j \sum_p f_{2,p,d}^{i,j} \cdot \delta_{p,a} \qquad \forall a, d \quad (5)$$

Path cost and link cost relationship

$$C_{p,d}^{i,j} = \sum_a C_{a,d}(v_{a,d}, c_{a,d}) \cdot \delta_{p,a} \qquad \forall i, j, d, p \quad (6)$$

When it comes to long-term traffic strategy evaluation, for example, toll pricing, a conventional approach is to apply user equilibrium principle to capture the route choice of network users on each days with the mean capacity value for each link. However, this mean-value and perfect information approach over-estimates the capacity of user travel time perception and the accuracy of traffic information provision. More importantly, the existing model cannot represent and explain the actual disequilibrium conditions within each day.

4. System optimal flow assignment model

In this section, we formulate a day-dependent SO flow assignment model as

Objective function:

$$\min z = \sum_d \sum_i \sum_j \sum_p \left((f_{p,d}^{i,j}) \times C_{p,d}^{i,j} \right) \tag{7}$$

subject to constraints (2) – (6).

Similarly, a steady-state SO flow assignment model can be written as

Objective function:

$$\min z = \sum_d \sum_i \sum_j \sum_p \left((\overline{f}_p^{i,j}) \times C_{p,d}^{i,j} \right) \tag{8}$$

where $\overline{f}_p^{i,j}$ is the same path flows over multiple days. That is,

$$\overline{f}_p^{i,j} = f_{2,p,d=1}^{i,j} = f_{2,p,d=2}^{i,j} = \cdots = f_{2,p,d=D}^{i,j}$$

The proposed objective function is also subject to constraints (2) – (6).

5. Effectiveness of information provision under stochastic capacity

To demonstrate the insights that can be derived from the proposed models, an illustrative example is presented, which shares the simple experimental settings shown as Fig. 1. The optimization model is implemented using Visual Basic for Applications (VBA) and Excel's built-in solver. Stochastic capacity is generated based on a calibrated capacity distribution for 100 days. Three solution strategies are summarized and briefly compared below, they are

- Perfect information (PI) solution, where the market penetration of (real-time) information provision is 100%), and a different user equilibrium flow distribution can be obtained every day.
- Expected value (EV) solution, where the solution which assumes that the volumes (flow distribution) are the same every day and expected travel times on different (used) routes are the same.
- Day-dependent SO solution, where different SO flow distribution can be obtained every day.

In Fig. 4, the average travel time are plotted as a function of different demand levels. The travel times of perfect information solutions are always higher than those of SO solutions, and lower than those of expected value solutions. This is intuitive because users can reduce their travel times by obtaining and utilizing perfect information, while the SO solution shows the lower bound of system-wide travel time. Fig. 4 also shows that, the increase in demand level would reduce the travel time difference between the perfect information solution and the expected value solution when demand grows to a specific level (e.g. 6000 veh/h).

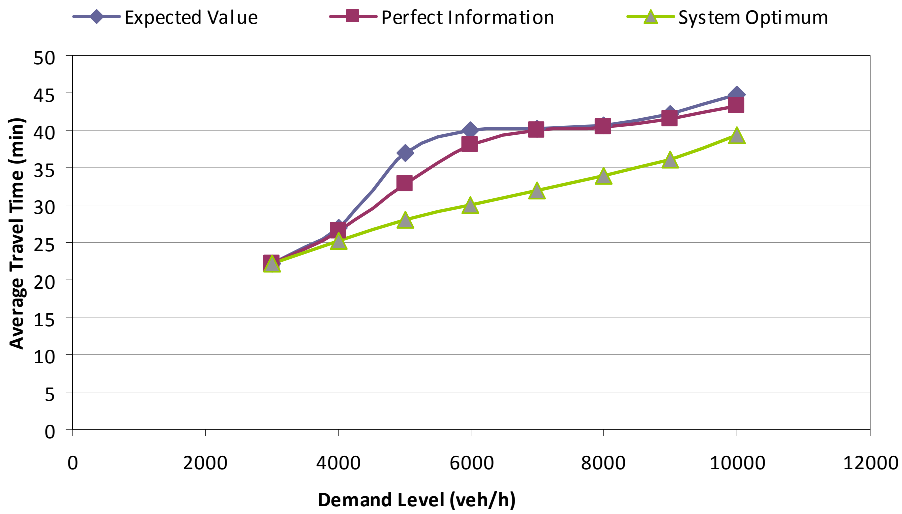

Fig. 4. Average of travel time vs. demand level.

Value of information is defined in this paper as the ratio of average travel time improvement due to information provision to average travel time under perfect information, that is

$$VOI = \frac{TT_{EV} - TT_{PI}}{TT_{PI}} \tag{9}$$

It can be used as a quantitative measure of how much the information provision improves the system performance with respect to travel time. Fig. 5 shows the results of value of information for free-flow travel time, ranging from 0 to 60 minutes at 4000, 5000 and 6000 veh/h demand levels. The plot demonstrates that, as free-flow travel time difference increases (that is the alternative route becomes less attractive compared to the primal route), the value of information will also increase until the value of information reaches its maximum value. After that the value of information will decrease if free-flow travel time difference continues to increase. It is interesting to note that the peaks of the value of information function shift to right, as demand level increase, meaning that maximum value of information is obtainable under more congested condition due to higher demand.

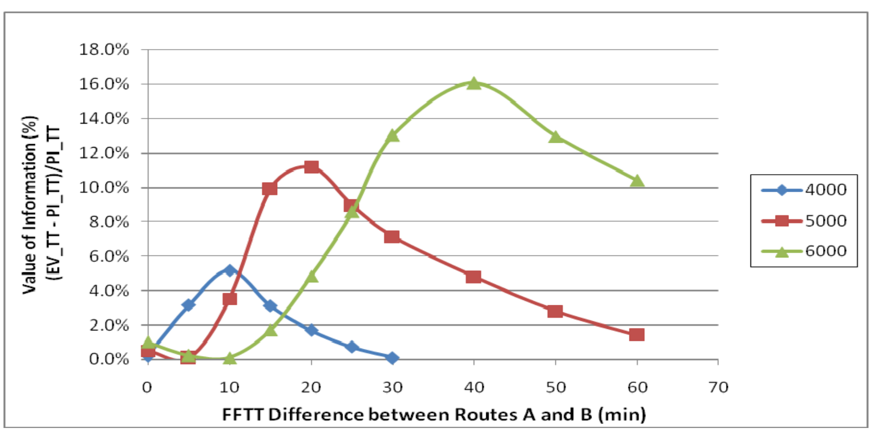

Fig. 5. Value of information as a function of FFTT difference and demand level.

6. Concluding remarks

Aiming to capture day-dependent travel times under stochastic (day-varying) capacities, and evaluate the potential benefits of information provision strategies, this paper proposes a new type of static traffic assignment method, which differentiates two user classes with respect to stochastic travel times over different days. Two types of SO models, steady-state SO and day-dependent-SO are also formulated for stochastic capacity. A simple numerical example has been presented to demonstrate the application of the model to quantifying the value of information. Future research will focus on (a) testing the model performance on full-scale networks and real datasets, and (b) adapting the model framework to day-dependent/invariant toll applications.

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