Deconstructing utility in activity-travel choice models

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Motivation:utility for everyone

- **General**: people choose what they like most, and people is different. So everything fits in;

  U(travel cost, travel time, income, gender, frequency, period, seats, other activities, family structure, etc.)?

- **Specific**: Is it better a quadratic or a linear? Or Cost/income? Better fit? Flexibility?
- **Philosophical**: Shall we let the data talk?

- Beginning: Where does utility in discrete travel choice come from?

$$\underset{X,j}{Max}\, U(X, Q_j)$$
$$\sum P_i X_i + c_j \le I$$
$$J \in M$$

$$\rightarrow$$

$$\underset{X}{Max}\, U(X, Q_j)$$
$$\sum P_i X_i \le I - c_j$$

$$\rightarrow \quad X^*(P, Q_j, I - c_j)$$

conditional demands

$$U\left[X^*(P, Q_j, I - c_j), Q_j\right] = V(P, Q_j, I - c_j) \equiv V_j$$

Conditional Indirect Utility Function (truncated)

$$\underset{j \in M}{Max}\, V(P, Q_j, I - c_j) \rightarrow \quad V_k \,?\, V_L \quad \forall_{k,L \in M}$$

$$MUI = \lambda = \frac{\partial V}{\partial I} = -\frac{\partial V_j}{\partial c_j}$$

Marginal Utility of Income

$$SVq_{ji} = \frac{\partial V_j / \partial q_{ji}}{\partial V_j / \partial I}$$

Subjective Values

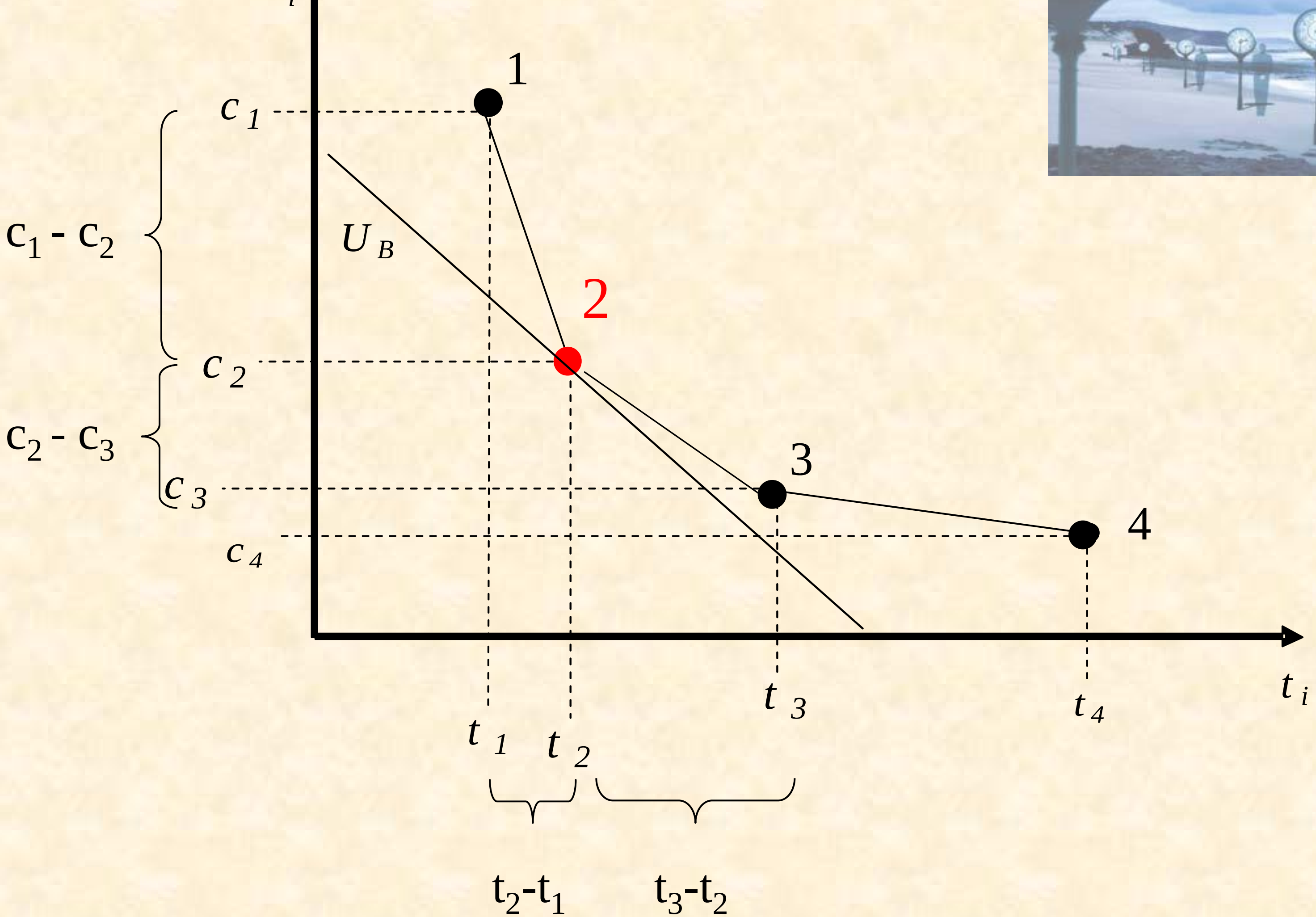
Some corollaries

- Unless V_i is linear, income is income, not a surrogate for either taste or preferences.

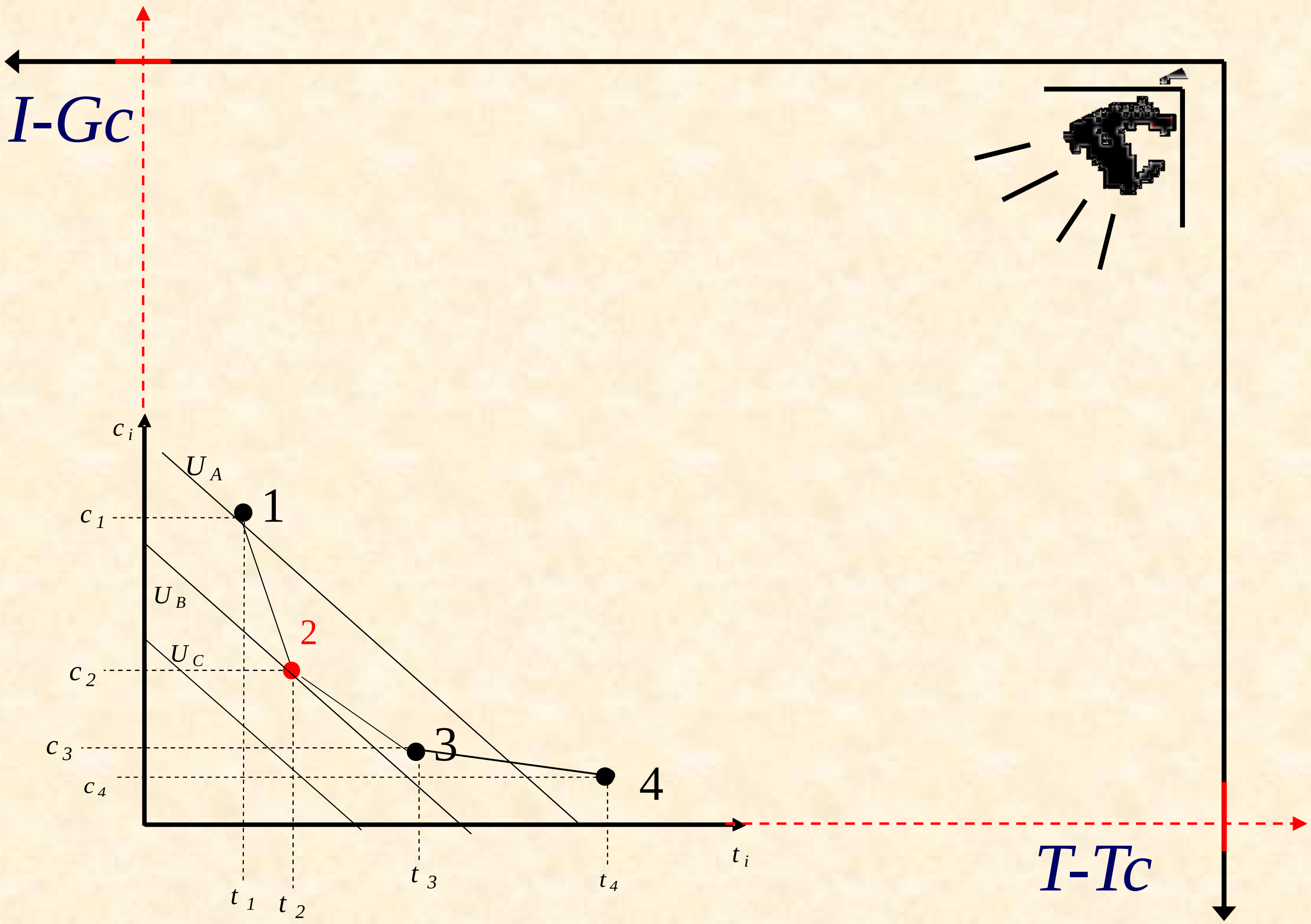
- $I\text{-}c_j$ in V $\rightarrow$ significant second order term in c_i implies that MUI depends on Income: income effect in travel choice.

Introducing travel time (and its value).

Perspective 1

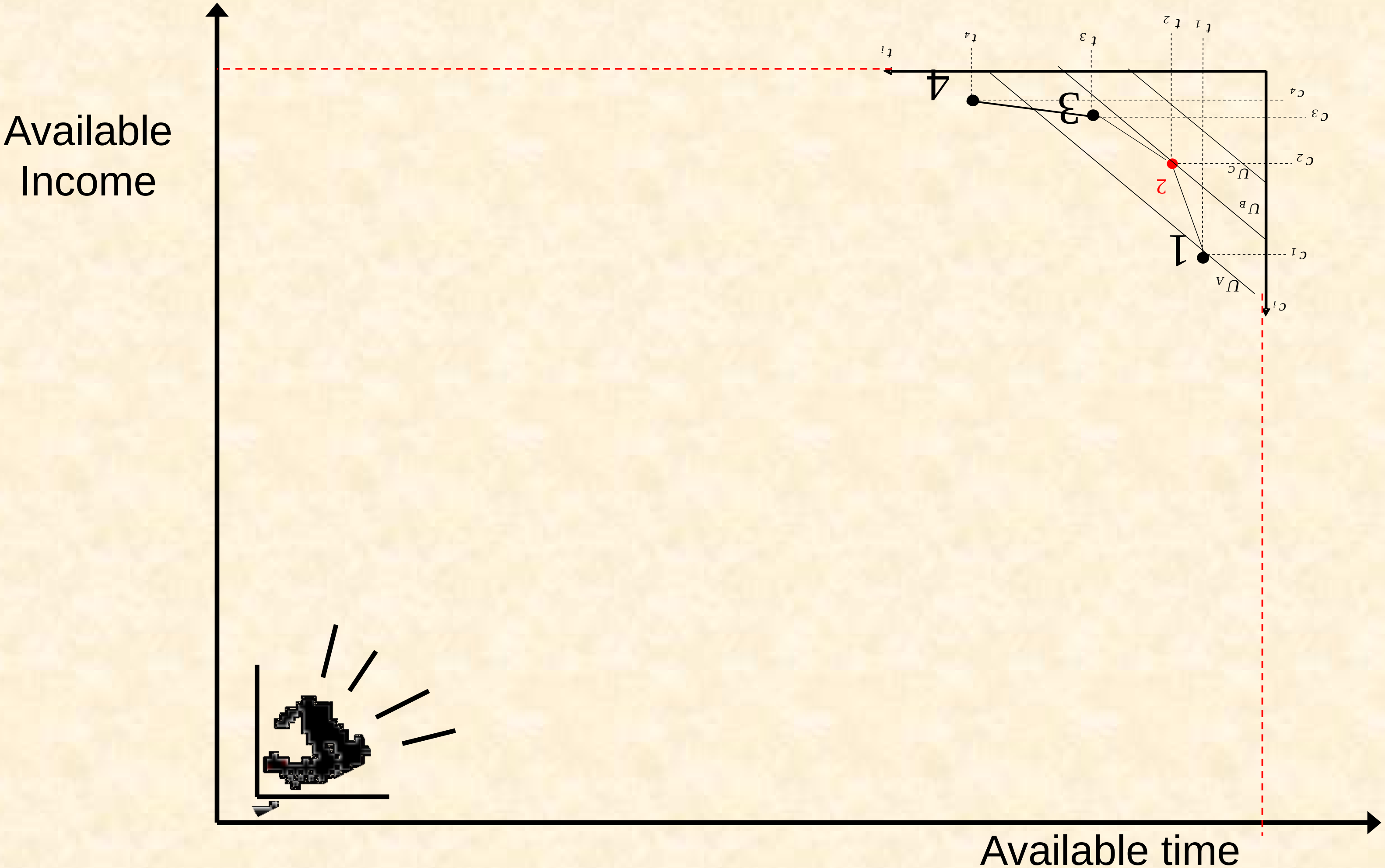


Perspective 1



Diminish travel time by paying more…

Perspective 2



… or increase discretionary free time by diminishing available income

Goods-Leisure framework (Train and McFadden;1978)

The individual behaves as if:

$$\left.\begin{array}{l} Max_{S.to}\ U(G, L) \\ G + c_i \le wW \\ L + W + t_i = \tau \\ i \in M \end{array}\right\}$$

$$U\left[wW - c_i,\ (\tau - W - t_i)\right]$$

$$\frac{\partial U}{\partial W} = 0 \quad \Rightarrow \quad W^*(c_i, w, t_i)$$

$$\therefore V_i = U\left[c_i, w, t_i\right]$$

Discrete analogy of Becker (1965)

Corollaries

- $SVTTS = w = VoT$
- Justifies c_i/w as a variable in V_i
- Implicit labor supply model
- If income is fixed,
  - c_i/g in V_i if c_i/I small
  - Use second order terms if c_i/I and/or $t_i/(\tau - W)$ non-negligible

The goods-activities framework

$$\begin{array}{l} Max \quad U(T, X) \\ subject \quad to \end{array}$$

Income constraint (λ)
Total time constraint (μ)
Tecnological constraints (κ)

Leads to

T*(….), X*(….)

U[T*(….), X*(….)]≡V(….)

DeSerpa's theory (1971)

Max $U(X, T)$

(1) $\Sigma P_i X_i = w T_w$ (λ)

(2) $\Sigma T_j = \tau$ (μ)

(3) $T_j \geq a_j X_i$ (κ_j)

- κ_j / λ : value of a time reduction in constrained activity j *(zero for leisure activities)*
- μ / λ : value of time as a resource (value of leisure)
- $(\partial U / \partial T_j) / \lambda$: value of assigning time to activity j (value of the marginal utility)

F.O.C. $\rightarrow$

a) $\kappa_j / \lambda = \mu / \lambda - (\partial U / \partial T_j) / \lambda$

b) $\mu / \lambda = w + (\partial U / \partial T_w) / \lambda$

Therefore…

b) Value of leisure = total value of work

a) Value of time reduction in travel =
value of doing something else – intrinsic value of travel

Corollaries

- Pleasent travel not enough for *SVTTS* to be negative
- Implicit solution for T_w
- Implicit equations for leisure activities

The goods-activities model (Jara-Díaz and Guerra, 2003)

Max $$U = \Omega T_w^{\theta_w} \prod_i T_i^{\theta_i} \prod_j X_j^{\eta_j}$$

subject to

$$I_f + wT_w - \sum_j P_j X_j \geq 0 \leftarrow \lambda$$

$$\tau - T_w - \sum_i T_i = 0 \leftarrow \mu$$

$$T_i - T_i^{Min.} \geq 0 \leftarrow \kappa_i \quad \forall i$$

$$X_j - X_j^{Min} \geq 0 \leftarrow \varphi_j \quad \forall j$$

Work, Leisure, Goods and Travel equations

$$T_w^* = \beta(\tau - T_c) + \alpha\frac{E_c}{w} + \sqrt{\left(\beta(\tau - T_c) + \alpha\frac{E_c}{w}\right) - (2\alpha + 2\beta - 1)(\tau - T_c)\frac{E_c}{w}}$$

$$T_i^* = \frac{\vartheta_i}{1-2\beta}\left(\tau - T_w^*\left(\frac{E_c}{w}, T_c\right) - T_c\right) \qquad \forall i \text{ not binding}$$

$$X_k^* = \frac{\gamma_k}{1-2\alpha}\frac{w}{P_k}\left(T_w^*\left(\frac{E_c}{w}, T_c\right) - \frac{E_c}{w}\right) \qquad \forall k \text{ not binding}$$

$$V = \tilde{\Omega} w^{1-2\alpha}\left(T_w^* - \frac{E_c}{w}\right)^{1-2\alpha}(\tau - T_w^* - T_c)^{1-2\beta} T_w^{*2\alpha+2\beta-1}\prod_{r\in R} T_r^{Min\,\vartheta_r}\prod_{j\in J} X_j^{Min.\gamma_j}$$

Corollaries

- $T_i (E_c, T_c, w)$ system looks like a reduced form of a "structural equations" model.
- Values of work, leisure, travel and *SVTTS* can be calculated
- $T_w (E_c, T_c, w)$ equation is a more complete labor supply equation (goods-leisure particular case)
- Change in time assignment (labor and leisure activities) can be predicted after changes in E_c and/or T_c

Conclusions

- Understanding utility as a **TCIUF** facilitates specification and interpretation
- Behind the **TCIUF** always is a system of activities and goods consumption equations
- Gross classification of activities:
  a. Those one would like to increase but can not because of time budget (leisure);
  b. Those one would like to decrease but can not because of technical constraints ;
  c. Work and others.
- For b-type activities, Value of reduction = value of doing something else + value of diminishing mandatory time assigned.
- Observed Time Use permits empirical estimations of these values of time using econometric models: transport (three decades), activities.
- Applications so far show that:
  - Value of work time can be positive or negative.
  - Value of leisure can be different from the wage rate.
  - Increasing available time can be more important than travel displeasure.
  - Better to use segments than include socio-demographic variables in *U*.

Motivation for further research

- Time assigned to work is a new Labor Supply model where the marginal utility of work can be different from zero.

- *A priori* classification of activities can be explored empirically and econometrically.

- Single period (cross-sectional) models may not account for potentially relevant time use related decisions (but...).

- *Necessary link with sociology, psychology and biology to further analyze results.*

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