Deconstructing utility in activity-travel choice models

Sergio R. Jara-Díaz
Universidad de Chile
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Motivation: utility for everyone

- General: people choose what they like most, and people is different. So everything fits in; $U(\text{travel cost, travel time, income, gender, frequency, period, seats, other activities, family structure, etc.})$?

- Specific: Is it better a quadratic or a linear? Or Cost/income? Better fit? Flexibility?

- Philosophical: Shall we let the data talk?

- Beginning: Where does utility in discrete travel choice come from?
\[ \text{Max} U_{x,j} (X, Q_j) \]
\[ \sum P_i X_i + c_j \leq I \quad J \in M \]

\[ \text{Max} U_X (X, Q_j) \]
\[ \sum P_i X_i \leq I - c_j \]

\[ X^* (P, Q_j, I - c_j) \] conditional demands

\[ U (X^* (P, Q_j, I - c_j), Q_j, I - c_j) \equiv V (P, Q_j, I - c_j) \equiv V_j \]

Conditional Indirect Utility Function (truncated)

\[ \text{Max} V_{j \in M} (P, Q_j, I - c_j) \rightarrow V_k \equiv V_L \quad \forall k, L \in M \]

\[ \text{MUI} = \lambda = \frac{\partial V}{\partial I} = -\frac{\partial V_j}{\partial c_j} \]

Marginal Utility of Income

\[ \text{SV} q_{ji} = \frac{\partial V_j / \partial q_{ji}}{\partial V_j / \partial I} \]

Subjective Values
Some corollaries

- Unless \( V_i \) is linear, income is income, not a surrogate for either taste or preferences.

- \( l - c_j \) in \( V \) → significant second order term in \( c_i \) implies that MUI depends on Income: income effect in travel choice.
Introducing travel time (and its value).

Perspective 1
Perspective 1

Diminish travel time by paying more...
Perspective 2

... or increase discretionary free time by diminishing available income
Goods-Leisure framework (Train and McFadden; 1978)

The individual behaves as if:

\[
\begin{align*}
\text{Max}_{S.t.o} & \quad U \quad G, L \\
G + c_i & \leq wW \\
L + W + t_i & = \tau \\
i & \in M
\end{align*}
\]

\[
\begin{align*}
U & \quad wW - c_i \quad -W - t_i \\
\frac{\partial U}{\partial W} & = 0 \quad \Rightarrow \quad W^* \quad i, w, t_i \\
\therefore V_i & = U \quad i, w, t_i
\end{align*}
\]

Discrete analogy of Becker (1965)
Corollaries

- $SVTTS = w = VoT$

- Justifies $c_i/w$ as a variable in $V_i$

- Implicit labor supply model

- If income is fixed,
  - $c_i/g$ in $V_i$ if $c_i/l$ small
  - Use second order terms if $c_i/l$ and/or $t/(T-W)$ non-negligible
The goods-activities framework

\[ \text{Max } U \left( T^*, X^* \right) \]

subject to

Income constraint (\( \lambda \))
Total time constraint (\( \mu \))
Tecnological constraints (\( \kappa \))

Leads to

\[ T^*(....), \quad X^*(....) \]

\[ U[T^*(....), X^*(....)] \equiv V(....) \]
DeSerpa’s theory (1971)

Max \( U(X, T) \)

(1) \[ \Sigma P_i X_i = w T_w \quad (\lambda) \]

(2) \[ \Sigma T_j = \tau \quad (\mu) \]

(3) \[ T_j \geq a_j X_i \quad (\kappa_j) \]

- \( \kappa_j / \lambda \): value of a time reduction in constrained activity \( j \) (zero for leisure activities)

- \( \mu / \lambda \): value of time as a resource (value of leisure)

- \( (\partial U / \partial T_j) / \lambda \): value of assigning time to activity \( j \) (value of the marginal utility)
F.O.C. →

\[ a) \frac{\kappa_j}{\lambda} = \frac{\mu}{\lambda} - \frac{(\partial U/ \partial T_j)}{\lambda} \]

\[ b) \frac{\mu}{\lambda} = w + \frac{(\partial U/ \partial T_w)}{\lambda} \]

Therefore...

b) Value of leisure = total value of work

a) Value of time reduction in travel = value of doing something else – intrinsic value of travel
Corollaries

- Pleasant travel not enough for SVTTS to be negative
- Implicit solution for $T_w$
- Implicit equations for leisure activities
The goods-activities model (Jara-Díaz and Guerra, 2003)

Max \[ U = \Omega T_w^{\theta_w} \prod_i T_i^{\theta_i} \prod_j X_j^{\eta_j} \]

subject to

\[ I_f + wT_w - \sum_j P_j X_j \geq 0 \leftarrow \lambda \]

\[ \tau - T_w - \sum_i T_i = 0 \leftarrow \mu \]

\[ T_i - T_i^{\text{Min.}} \geq 0 \leftarrow \kappa_i \ \forall i \]

\[ X_j - X_j^{\text{Min}} \geq 0 \leftarrow \phi_j \ \forall j \]
Work, Leisure, Goods and Travel equations

\[ T_w^* = \beta \left( -T_c + \alpha \frac{E_c}{w} \right) + \sqrt{\left( \beta \left( -T_c + \alpha \frac{E_c}{w} \right) \right)^2 - \left( \alpha + 2 \beta - 1 \right) \left( -T_c + \frac{E_c}{w} \right)} \]

\[ T_i^* = \frac{\theta_i}{1 - 2\beta} \left( \tau - T_w^* \left( \frac{E_c}{w}, T_c \right) - T_c \right) \quad \forall i \text{ not binding} \]

\[ X_k^* = \frac{\gamma_k}{1 - 2\alpha} \frac{w}{P_k} \left( T_w^* \left( \frac{E_c}{w}, T_c \right) - \frac{E_c}{w} \right) \quad \forall k \text{ not binding} \]

\[ V = \tilde{\omega} w^{1-2\alpha} \left( T_w^* - \frac{E_c}{w} \right)^{1-2\alpha} \left( -T_w^* - T_c \right)^{2\beta} T_w^{*2\alpha+2\beta-1} \prod_{r \in R} T_r^{\text{Min} \theta_r} \prod_{j \in J} X_j^{\text{Min} \gamma_j} \]
Corollaries

- $T_i(E_c, T_c, w)$ system looks like a reduced form of a “structural equations” model.

- Values of work, leisure, travel and SVTTS can be calculated

- $T_w(E_c, T_c, w)$ equation is a more complete labor supply equation (goods-leisure particular case)

- Change in time assignment (labor and leisure activities) can be predicted after changes in $E_c$ and/or $T_c$
Conclusions

• Understanding utility as a TCIUF facilitates specification and interpretation
• Behind the TCIUF always is a system of activities and goods consumption equations
• Gross classification of activities:
  a. Those one would like to increase but can not because of time budget (leisure);
  b. Those one would like to decrease but can not because of technical constraints;
  c. Work and others.
• For b-type activities, Value of reduction = value of doing something else + value of diminishing mandatory time assigned.
• Observed Time Use permits empirical estimations of these values of time using econometric models: transport (three decades), activities.
• Applications so far show that:
  – Value of work time can be positive or negative.
  – Value of leisure can be different from the wage rate.
  – Increasing available time can be more important than travel displeasure.
  – Better to use segments than include socio-demographic variables in $U$. 
Motivation for further research

- Time assigned to work is a new Labor Supply model where the marginal utility of work can be different from zero.

- A priori classification of activities can be explored empirically and econometrically.

- Single period (cross-sectional) models may not account for potentially relevant time use related decisions (but...).

- Necessary link with sociology, psychology and biology to further analyze results.
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