

A Methodology for Achieving Internal Consistency in the Dallas-Fort Worth  
Travel Demand Model through Improvements in Traffic Assignment

Submitted to

3<sup>rd</sup> Conference on Innovations in Travel Modeling

May 9-12, 2010

Tempe, Arizona

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## **1 BACKGROUND**

The four-step travel demand models (TDM) have been widely used for forecasting the traffic flows on roadway networks. These models are able to answer many of the general questions that would be raised by planners and decision makers regarding the global effects of major roadway improvements on the regional travel patterns. However, these questions are getting more complicated and smaller in scale as the project selections become a more sensitive process due to factors such as limited available funds or increased public awareness and attention to the transportation issues. These analyses often times require using the regional TDM for comparing the benefits of each available alternative at a scale that could potentially be within the model's noise level. An alternate option is to use microscopic traffic simulation models for a corridor or a sub-area with the trip data from the regional four-step model as the demand input. The travel demand models, however, are constructed based on assumptions that do not necessarily match the basis on which the microscopic simulation models have been built (i.e., traffic control delay, volume and travel time relation, route choice, etc.). The North Central Texas Council of Governments (NCTCOG) has developed a methodology for achieving internal consistency in the Dallas-Fort Worth (DFW) travel demand model utilizing a conical volume-delay function (VDF) with integrated traffic control delay. This process involved the implementation of an iterative process with feedback loops and the application of convergence criteria which incorporated the network measures listed below:

- relative RMSE of AM peak period skim matrices used in trip distribution and the skims resulted from traffic assignment;
- maximum percent change in the cell-by-cell values of the AM peak period skims used in trip distribution and the skims resulted from traffic assignment;
- link volumes relative RMSE in two consecutive feedback loops; and
- ratio of maximum link volume difference over one lane capacity for each roadway functional classification.

The relative gap was set to  $1 \times 10^{-4}$  in the TDM application environment with maximum of 1000 iterations. The skims used in trip distribution were obtained by applying a constant weight of 75/25 to the skims used in the previous trip distribution step and the ones resulted from the traffic assignment. The model run is controlled by an in-house developed application that stops when all the preset limits for the above variables have been reached, with a maximum of 12 feedback loops. This paper describes the VDF that was utilized in the process and the results obtained from the model runs.

## **2 VOLUME-DELAY FUNCTION**

Several algorithms for solving the traffic assignment problem in a four-step travel demand model have been developed, each capable of producing results with different levels of accuracy. However, the VDF is one of the essential elements of

all the algorithms. The function used for a VDF must be continuous, monotone and increasing, and differentiable to ensure convexity, convergence and uniqueness of the solution, and must be defined for oversaturated regions as well.

## 2.1 CONGESTION DELAY

The VDF that was utilized in this analysis followed a conical form where the congested travel time is expressed as a function of the free-flow travel time and the conical shape variable. This function has been shown to satisfy the abovementioned criteria [1]. The general form of this function can be written as shown in Equation (1).

$$C_d = T_0 * (K_d - \{K_d \mid \frac{V}{C} = 0\}) \quad (1)$$

where:

$$K_d = \left( 1 + \sqrt{A\_CONICAL^2 * \left(1 - \frac{V}{C} + dx\right)^2 + B\_CONICAL^2} - A\_CONICAL * \left(1 - \frac{V}{C} + dx\right) - B\_CONICAL \right)$$

$$B\_CONICAL = \left( \left[ \frac{(2 * A\_CONICAL - 1)}{(2 * A\_CONICAL - 2)} \right] \right)$$

dx = horizontal shift in the VDF  
 $C_d$  = congestion delay  
 $T_0$  = free-flow travel time

The total congested link travel time is equal to the sum of  $C_d$  and  $T_0$ . The above equation has been slightly modified to incorporate a horizontal shift, applied based on the link functional classification, and a vertical correction shift to the function. These shifts have been applied in some other implementations by introducing a multiplier that is applied to the  $V/C$  ratio [2].

## 2.2 SIGNALIZED INTERSECTION DELAY

The signal delays in four-step travel demand models can be either implemented as constants, as a function of the volume to capacity ratio, or eliminated. The delays can be either incorporated in the VDF or reflected in the adjusted link free-flow speed. In this implementation the signal delays were included in the VDF based on the uniform delay component of the Webster's signal delay, shown in Equation (2) [3].

$$d_s = \frac{C(1 - \frac{q}{C})^2}{2(1 - \frac{q}{s})} + \frac{(\frac{q}{C})^2}{2q(1 - \frac{q}{C})} - 0.65 \left(\frac{C}{q^2}\right)^{1/3} \left(\frac{C}{q^2}\right)^{(2+5g/C)} \quad (2)$$

where:

$d_s$  = total signalized delay (seconds per vehicle)

$C$  = signal cycle-length (seconds)

$g$  = effective green time (seconds)

$q$  = approach volume (vph)

$c$  = approach capacity (vph) =  $(g/C).s$

$s$  = approach saturation flow rate (vph)

The first term of this equation (uniform delay,  $d_{su}$ ) can be rewritten as follows:

$$d_{su} = \frac{C(1-\frac{g}{C})^2}{2(1-\frac{q}{s})} = \frac{C(1-\frac{(C-r)}{C})^2}{2(1-\frac{q}{s})} = \frac{(C-g)^2}{2C(1-\frac{q}{s})} = \frac{r^2}{2C(1-\frac{q}{s})}$$

(3)

where :

$r$  = approach red time (seconds)

The signal cycle length has been calculated through a rather simple formulation based on the priority of the intersecting roadways, as shown in Equation (4) [4].

$$C_j = C_s + K_s n_j \sum w_{ij} \quad (4)$$

where:

$C_j$  = cycle length at intersection j (seconds)

$C_s$  = signal cycle constant (seconds)

$K_s$  = cycle-length multiplier

$n_j$  = number of links ending at node j

$w_{ij}$  = weight assigned to the approach link ij, as follows:

- 0, centroid connectors
- 2, collectors
- 3, minor arterials
- 4, major arterials
- 5, freeway and expressways

The approach red time has also been estimated based on the link priorities as shown in Equation (5) [4].

$$r_{ij} = C_r^k C_j \left(1 - \frac{n_j w_{ij}}{2 \sum w_{ij}}\right) \quad (5)$$

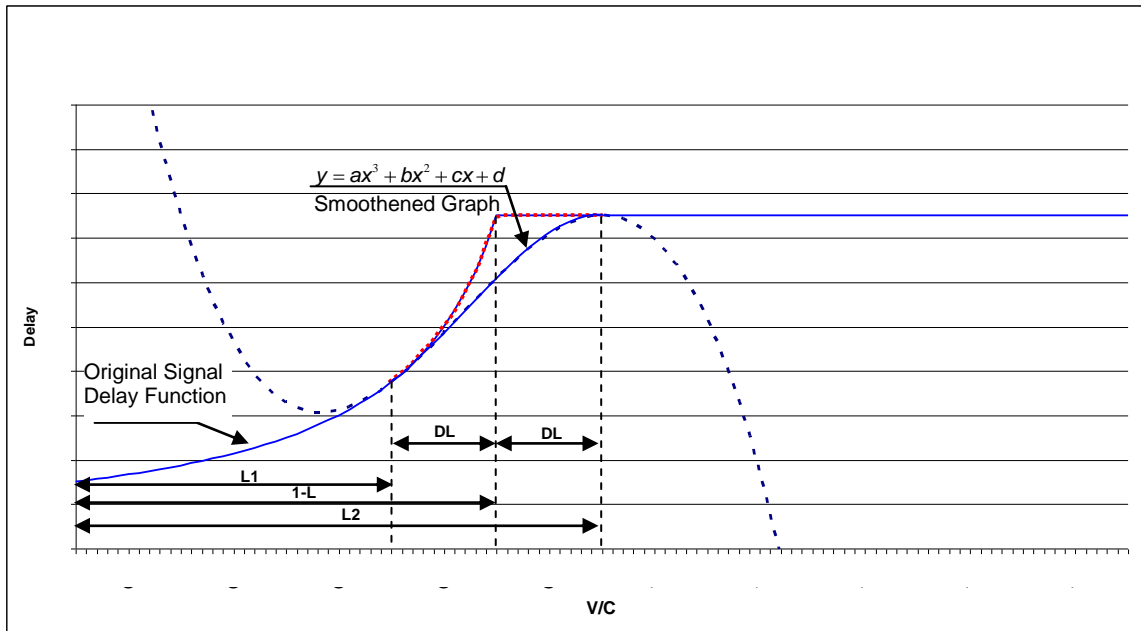
where:

$C_r^k$  = red time constant for functional classification  $k$ ,  $k = 1, 2, 3, 4, 6, 7$

The delay function in Equation (3) needs to be modified as in Equation (6) to prevent the denominator from becoming zero when  $q$  approaches  $s$ .

$$d_{su} = \frac{r_{ij}^2}{2C[\max(1 - \frac{q}{s}, L)]} \quad (6)$$

However, this equation does not satisfy the VDF requirements as it is neither monotone nor differentiable. Therefore, a second adjustment had to be made to replace the non-differentiable segment of the function with a smooth graph, as shown in **Figure 1**. The variables shown in **Figure 1** are model input except the multipliers of the smoothed graph that are calculated in the VDF for each link.



**Figure 1** Modified signalized intersection delay function.

The modified signal delay function is constant when  $V/C \geq L2$ . However, since the congestion delay is already monotone, increasing, and differentiable its sum with this signal delay function will satisfy the requirements of a VDF.

### 2.3 UN-SIGNALIZED INTERSECTION DELAY

The Highway Capacity Manual (HCM) delay calculation for un-signalized intersections is a rather complicated process that requires all the turning movements at the intersection as an input. Therefore, a simple approach was chosen in this implementation where the delay is expressed as a multiplier of the volume to capacity ratio, as shown in Equation (7).

$$d_u = d_{\min} + d \cdot \left(\frac{V}{c}\right) \quad (7)$$

where:

$d_u$  = un-signalized approach delay (seconds)  
 $d_{\min}$  = minimum delay at un-signalized intersections (seconds)  
 $v$  = approach volume (vph)  
 $c$  = approach capacity (vph)

$d$  is a factor that is calculated based on the number of inbound and outbound links and the number of prohibited turning movements at the intersection as follows:

$$d = m \cdot \left[ \frac{nk - w - p}{2} \right] \quad (8)$$

where:

$n$  = number of inbound links  
 $m = 3$  seconds for yield and four-way stops  
6 seconds for two-way stops  
 $k$  = number of outbound links  
 $w$  = number of two-way links  
 $p$  = number of turn prohibitions

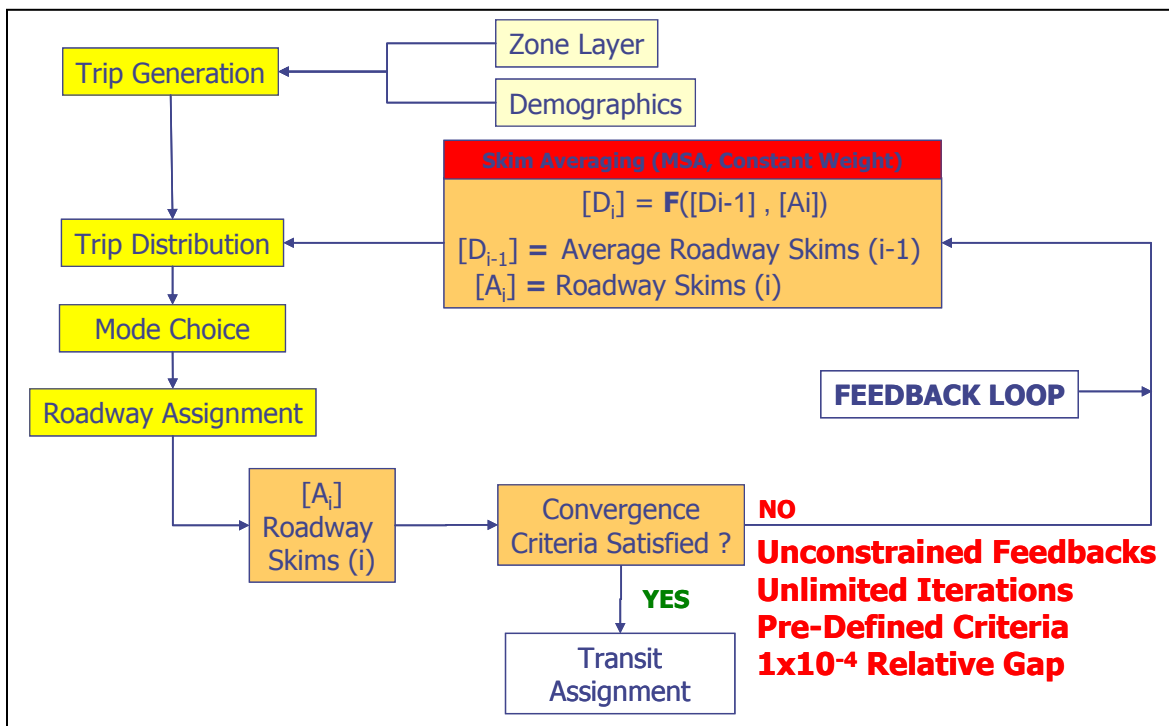
This function satisfies all the requirements of a VDF.

The total intersection delay is the sum of the congestion delay (Equation 1) and signal delay (Equation 6) or un-signalized delay (Equation 8).

### 3 TRAFFIC ASSIGNMENT CONVERGENCE CRITERIA

The traffic assignment convergence criteria are mostly defined in the assignment module of the modeling application as the relative gap and the maximum number of iterations, whichever is reached first. The smallest achievable relative gap is also a function of the method used for solving the assignment problem. The modeling process at NCTCOG uses the Frank Wolfe method for solving the User Equilibrium traffic assignment problem. It has been shown that the best relative gap achievable through the Frank Wolfe method is  $1 \times 10^{-4}$  [5]. The modifications

to the VDF were required to ensure the convergence of the final solution. Implementation of feedback loops is a common solution for reducing the model noise level. However, the question still remains that how many feedback loops should be run without increasing the run time to unacceptable levels. Therefore, the goal in this project was to associate a set of performance measures to the model run that would represent the changes in the results between each two consecutive feedback loops. The feedback loops in this implementation includes the application of a 75/25 constant weight to the skims, as shown in **Figure 2**. The selection of the constant weights was based on the previous work done on the TDM at NCTCOG [6]. The assignment result is considered converged only when all the criteria have been satisfied or a maximum number of 12 feedback loops have been reached.



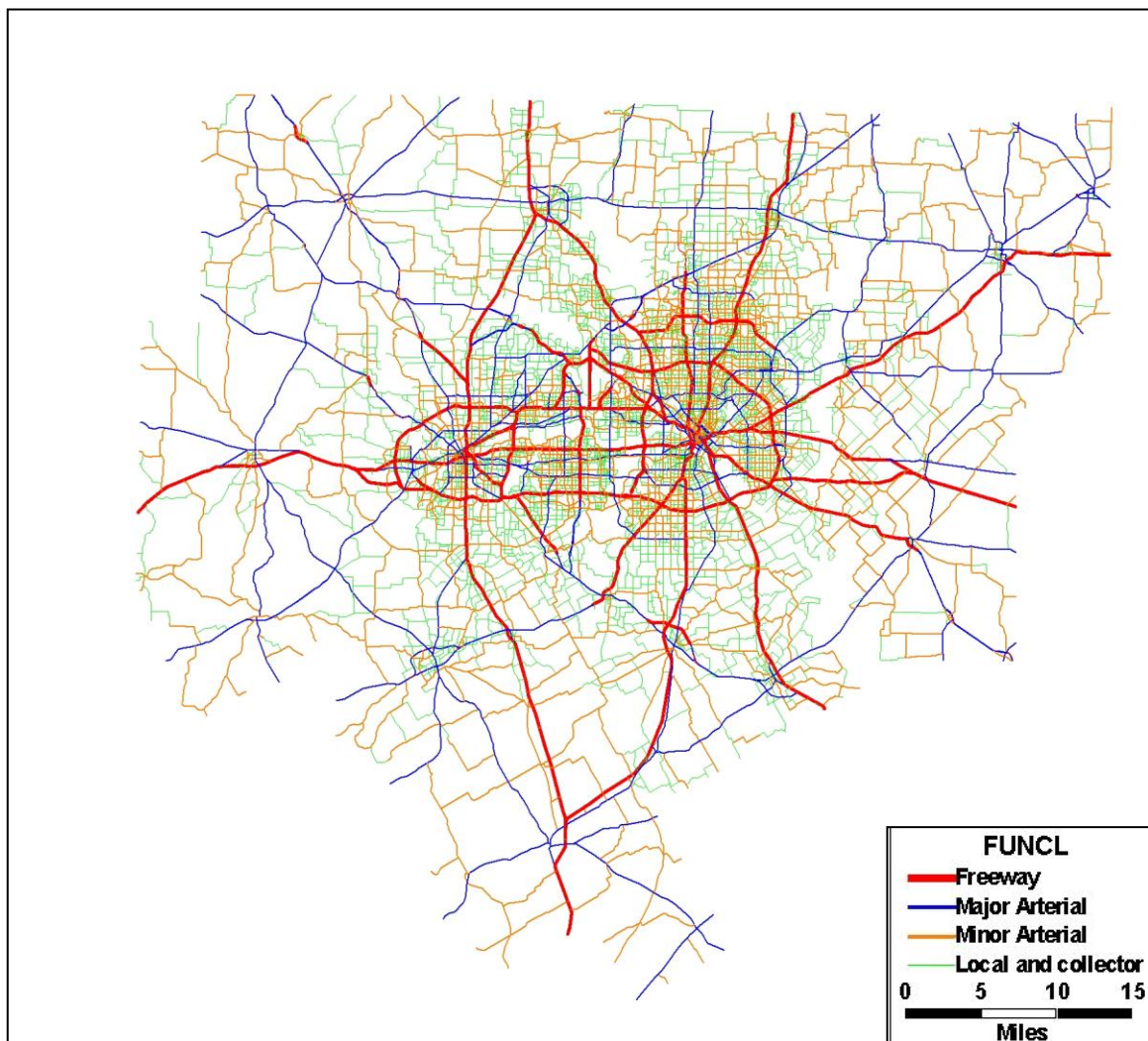
**Figure 2** Convergence check and feedback loop.

The performance measures and their corresponding values defined in the convergence criteria are as follows:

- Skim matrices RMSE  $\leq 1\%$
- Maximum cell-by-cell difference in skim matrices  $\leq 10\%$
- Link volume RMSE  $\leq 2\%$

- Ratio of link volume change over one-lane capacity
  - $\leq 15\%$ , freeways
  - $\leq 20\%$ , major arterials
  - $\leq 25\%$ , minor arterials
  - $\leq 25\%$ , local and collectors
  - $\leq 50\%$ , frontage roads and Ramps

The skim matrices used in this evaluation are the skims used for the trip distribution and the resultant of the traffic assignment. Therefore, the analysis results are not affected by the skim averaging in the feedback loop. However, the link volume comparisons are performed after two consecutive model runs. This application was run on the NCTCOG travel demand model roadway network, as shown in **Figure 3**.



**Figure 3** NCTCOG travel demand model roadway network.



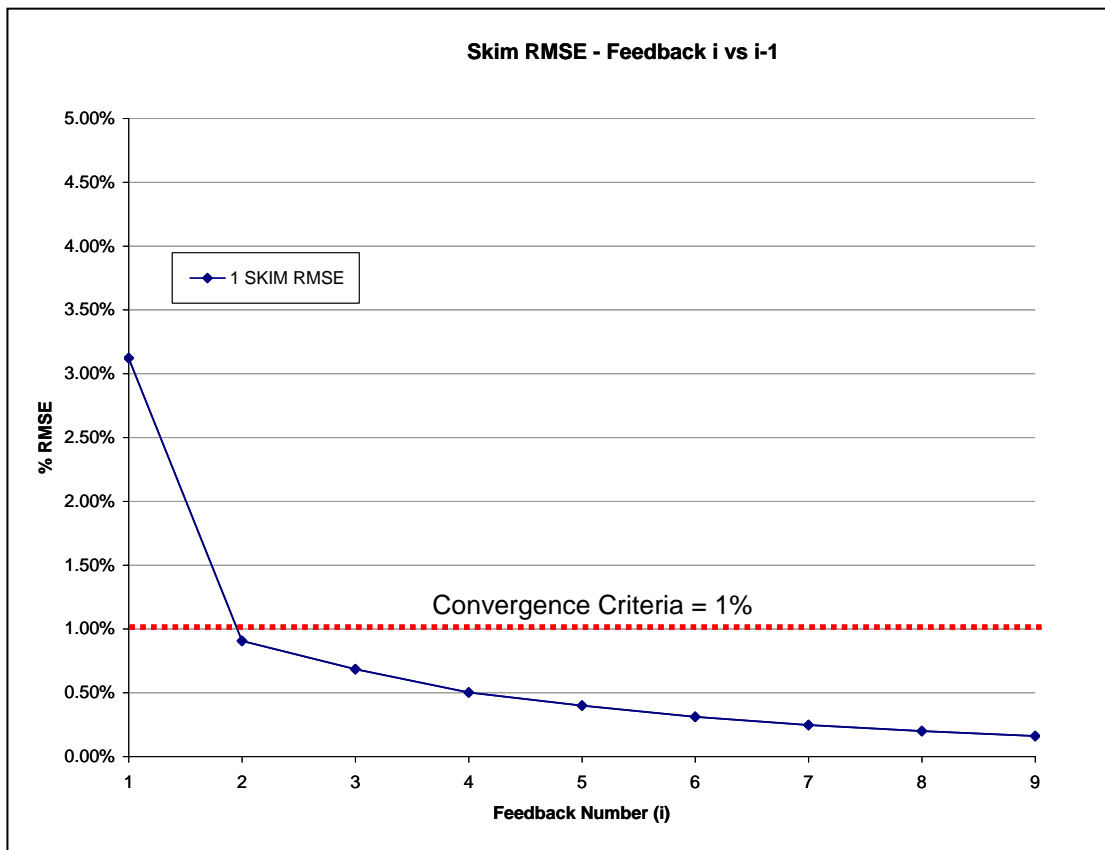
The feedback loop is controlled through a GISDK application that is called by an interface developed in Visual Basic.

#### 4 TRAFFIC ASSIGNMENT RESULTS

The above mentioned criteria were tested on the year 2004 and 2030 networks. It is apparent that the more congested the network the longer it takes for all the requirements to be achieved. The maximum of 12 feedback loops was selected based on the analysis results in year 2030. However, there are situations that we might have networks that are more congested than the current 2030 network. The convergence criteria for such networks might not be achieved within the maximum of 12 feedback loops. Therefore, the convergence measures will be evaluated for these models after the preset 12 feedback loops and the need for further analysis is identified. The convergence measures for the year 2004 modeling year are presented hereafter.

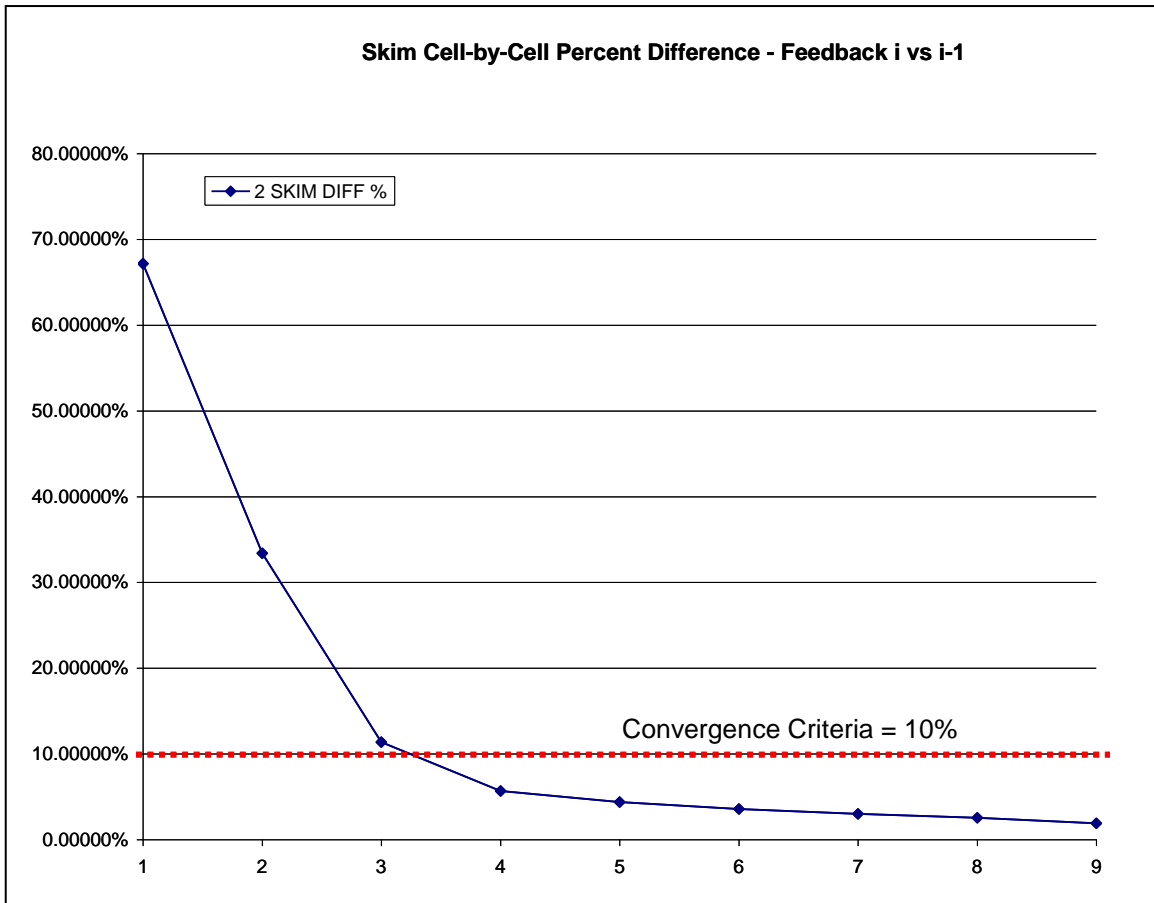
##### 4.1 CONVERGENCE MEASURES

The model runs on both the 2004 and 2030 networks indicated that the skim RMSEs are rather stable in every feedback loop and converge quickly, as shown in **Figure 4**. The acceptable RMSE is achieved after the second feedback loop.



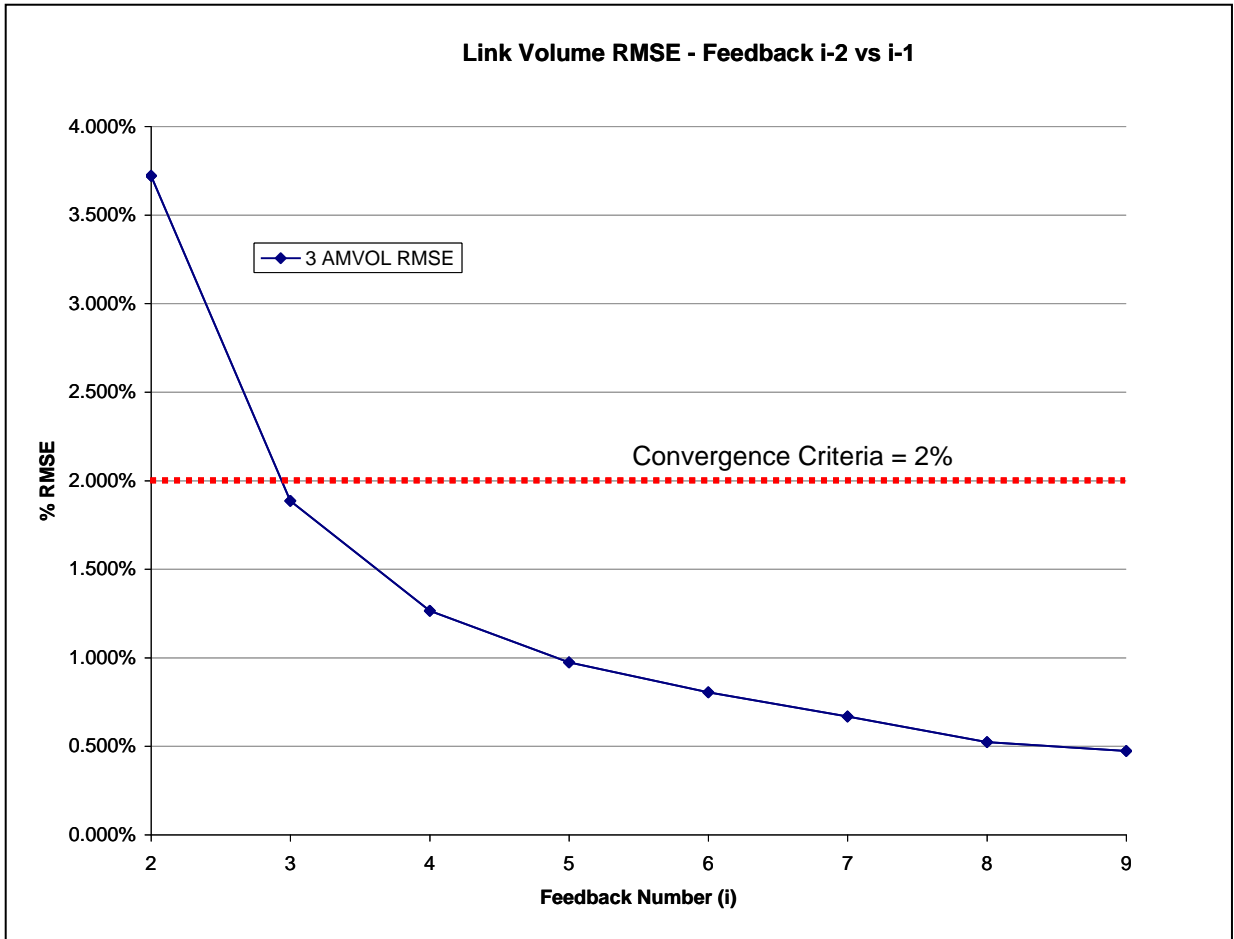
**Figure 4** Skim RMSE in two consecutive feedbacks.

The maximum cell-by-cell difference between each two consecutive skim matrices also shows a smooth convergence, as shown in **Figure 5**. However, the 2030 model runs showed a less smooth transition for this measure.



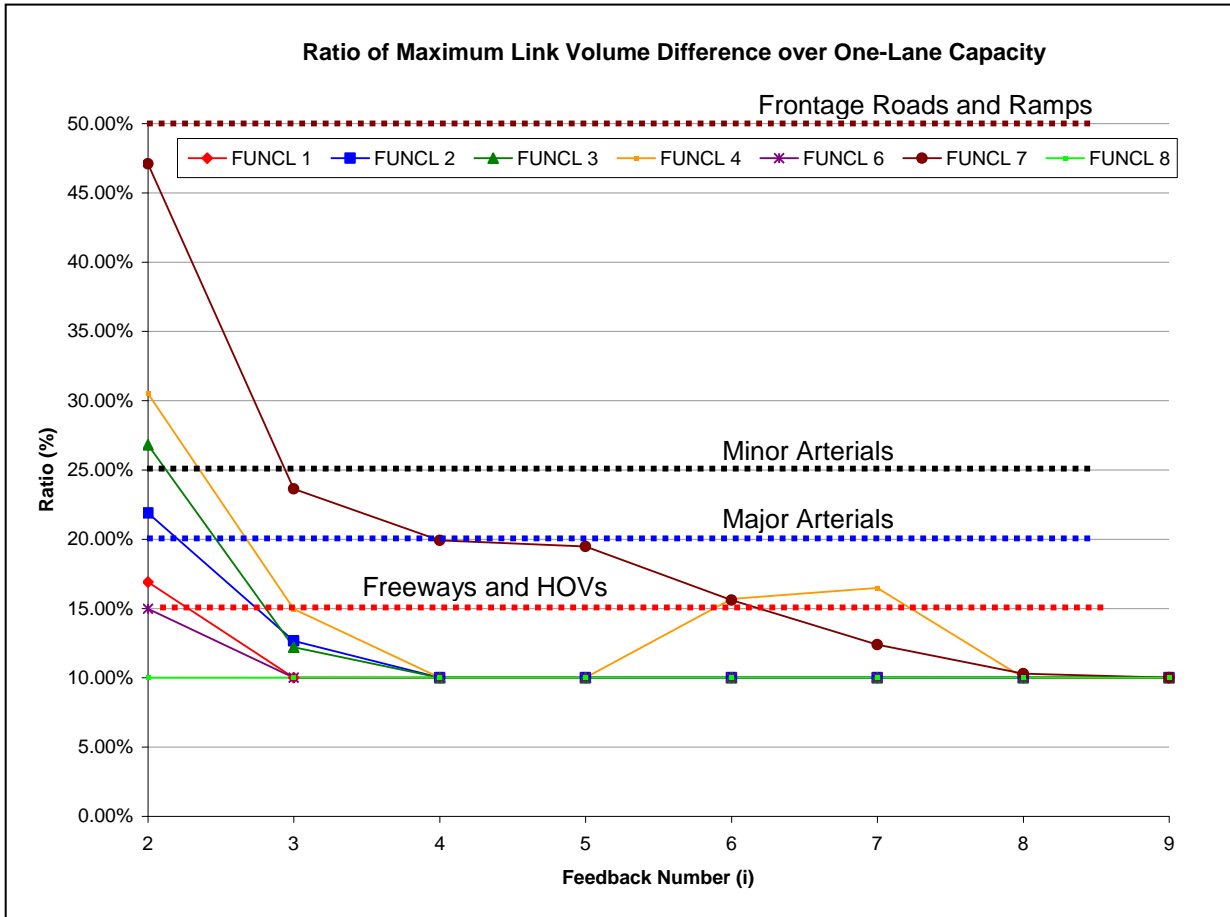
**Figure 5** Maximum skim cell-by-cell percent difference.

The travel demand model at NCTCOG is utilized in a wide range of applications including but not limited to alternative analysis. In many occasions the difference in the alternatives under consideration could be rather minor. Hence, it becomes a necessity that the link volumes stabilize during the feedback loops. Therefore, the change in the AM peak period link volume RMSE in two consecutive feedbacks was also included in the criteria, as shown in **Figure 6**. This measure has a clear physical significance as it relates to the maximum change in the link volumes between two consecutive feedbacks. The acceptable threshold is reached after the third feedback.



**Figure 6** Link volume RMSE in two consecutive feedbacks.

The last and probably the most difficult criteria to achieve is the threshold set for the ratio of the maximum link volume difference between two consecutive feedbacks over one-lane capacity per functional classification, as shown in **Figure 7**. The horizontal dotted lines on this graph show the convergence limits for each functional classification as identified in the legend. The common practice is to ensure that the possible range for the forecasted volumes is within one lane capacity for each functional classification. The results indicate that this measure follows a decreasing trend for most of the roadway types. However, in year 2004, the collectors show a slight jump between feedbacks 5 and 8.



**Figure 7** Ratio of maximum link volume difference over one-lane capacity.

## 5 CONCLUSIONS

The model runs performed on NCTCOGs TDM for year 2004 indicate that the conical volume delay function with integrated traffic control delay produces link volumes and travel times that are comparable with the available data for that year. The model run results for both 2004 and 2030 models show that the convergence criteria defined in the model application will be satisfied within a maximum 12 feedback loops. These criteria will reduce the model noise level and produce consistent results.

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