Handling Uncertainties and Unreliability in Data from Smart Sensors

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Outline

1. Sensor Technologies and Sensors Measurements

2. Location of Sensors to Maximize Variance Travel Time Reduction
   - The basic AVI Reader location models
   - Considering Link Failure
   - Considering Sensor failure

3. Conclusions and Further Research

Need for Sensors

Sensor Technologies and Sensors Measurements

Location of AVI Readers

Model AVI3

Model AVI4

Model AVI4 + Link Failure

Model AVI4 + Sensor Failure

Conclusions

We need sensors to detect and monitor traffic for:

- **Providing travel time and congestion information to motorists**
- **Providing data for traffic management so that traffic congestion is reduced**
- **Assisting in managing incidents and evacuations**
- **Providing data for trucking, transit, navigation, route guidance, transportation planning, …**

Sensor Technologies and Sensors Measurements

Sensor Technologies and Sensors Measurements

Location of AVI Readers

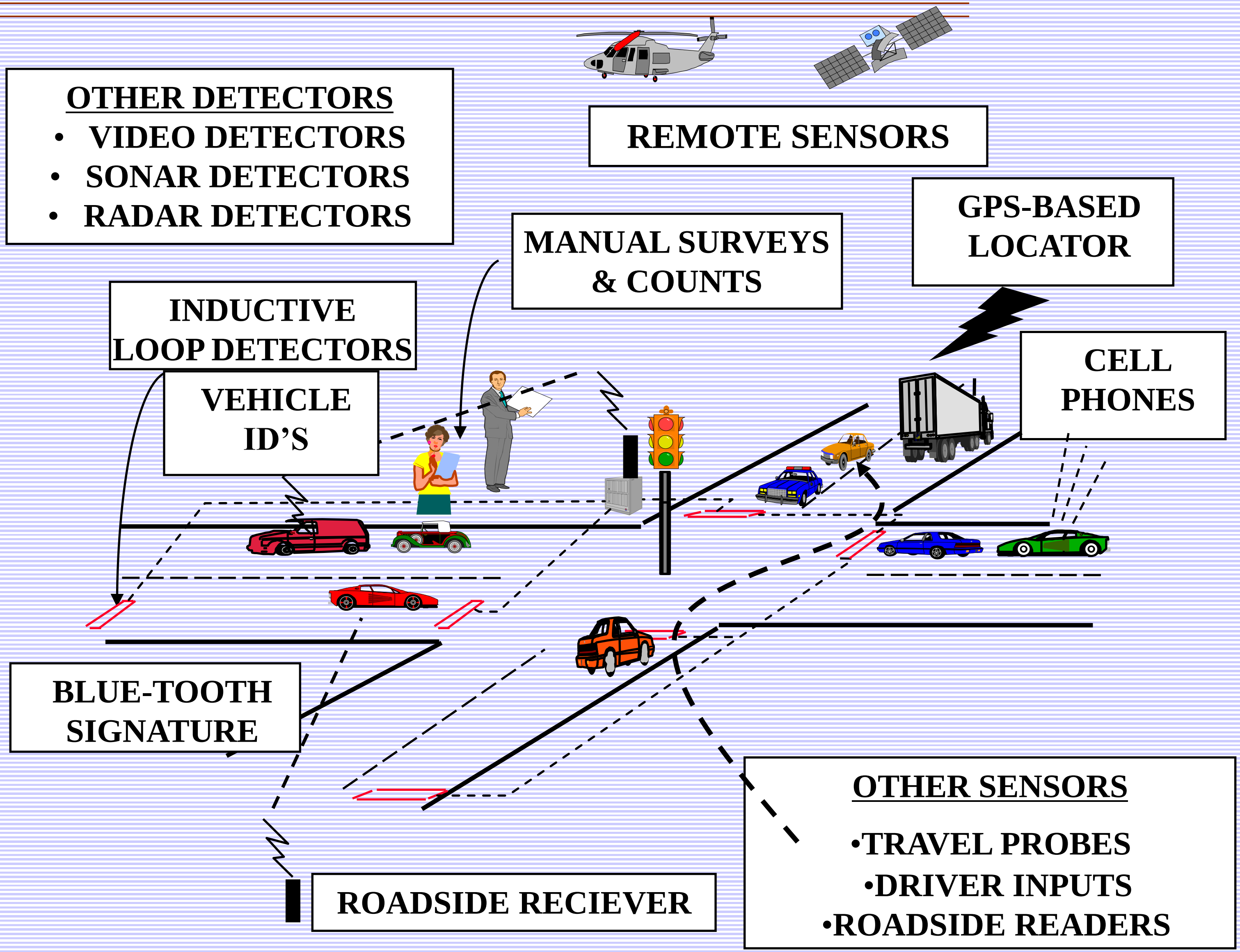
Model AVI3

Model AVI4

Model AVI4 + Link Failure

Model AVI4 + Sensor Failure

Conclusions



Locate AVI Readers to Monitor Travel Times

Sensor Technologies and Sensors Measurements

Location of AVI Readers

Model AVI3

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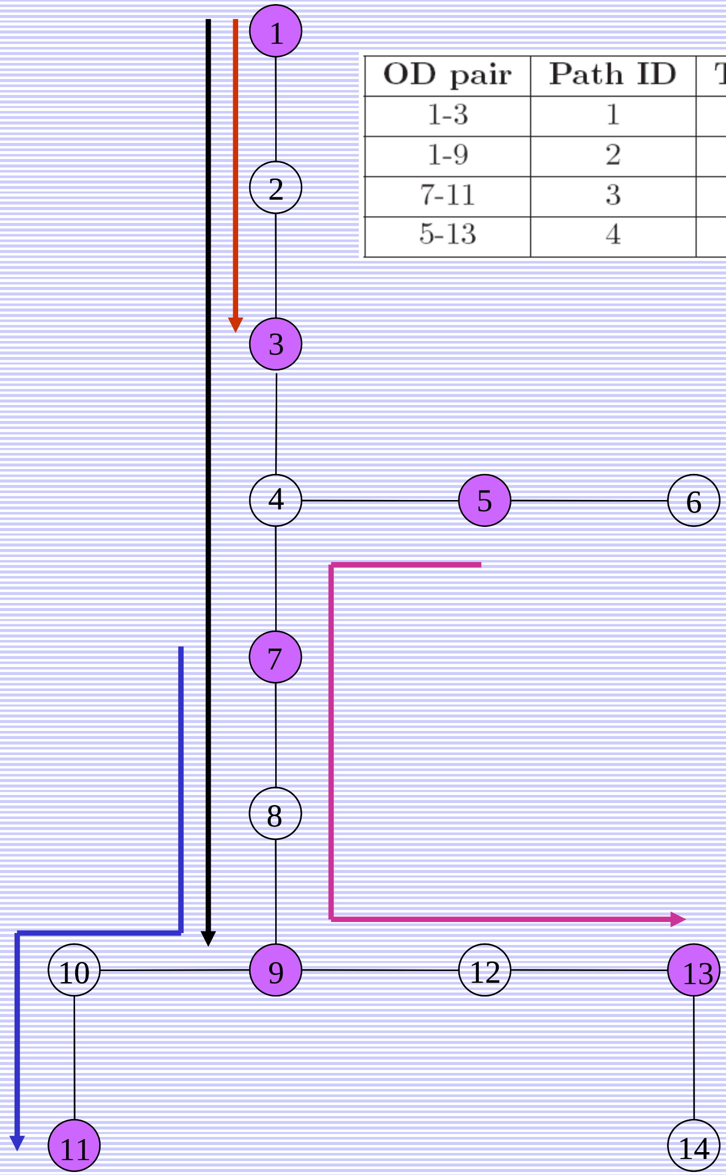
Model AVI4 + Sensor Failure

Conclusions

OD pair	Path ID	Trips	Links					
1-3	1	100	(1,2)	(2,3)				
1-9	2	10	(1,2)	(2,3)	(3,4)	(4,7)	(7,8)	(8,9)
7-11	3	10	(7,8)	(8,9)	(9,10)	(10,11)		
5-13	4	10	(5,4)	(4,7)	(7,8)	(8,9)	(9,12)	(12,13)



Locate AVI Readers to Monitor Travel Times

Sensor Technologies and Sensors Measurements

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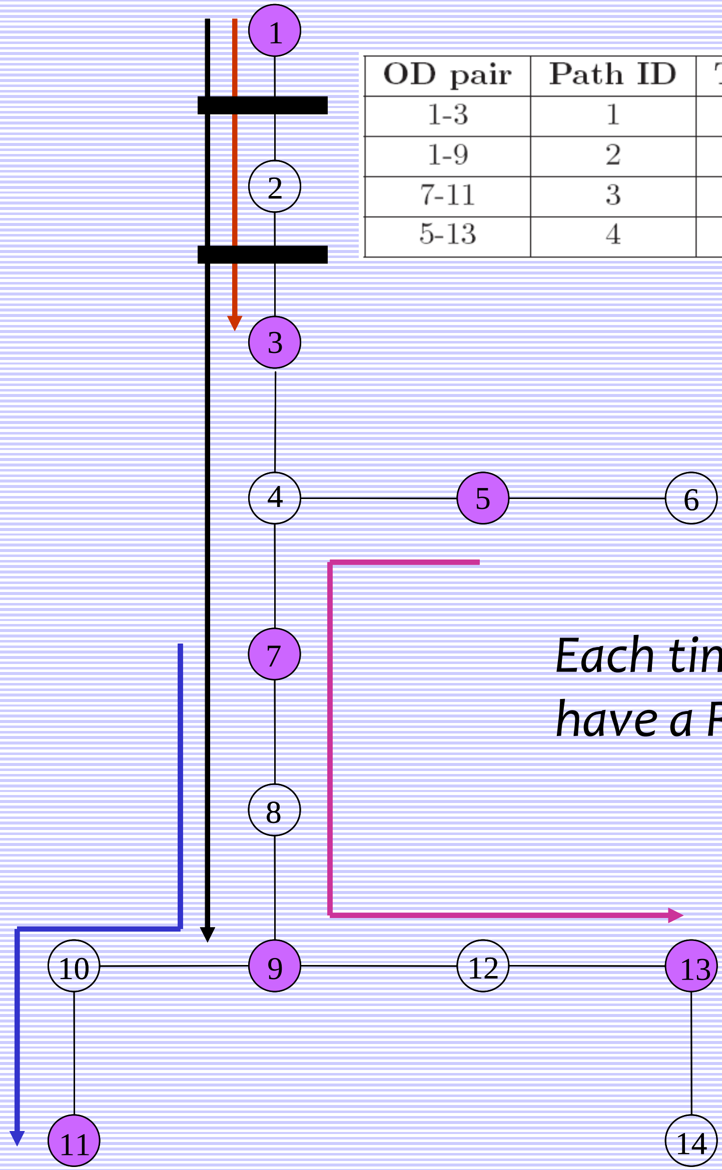

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Each time a vehicle passes two sensors we have a READING

Locate AVI Readers to Monitor Travel Times

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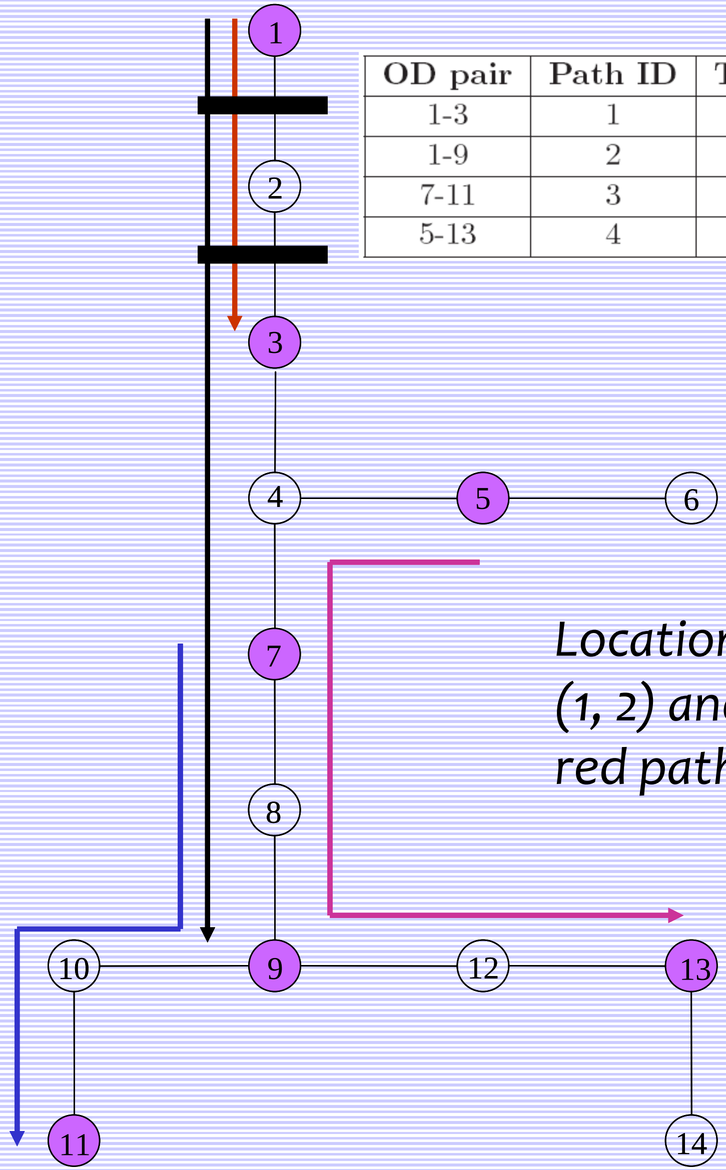
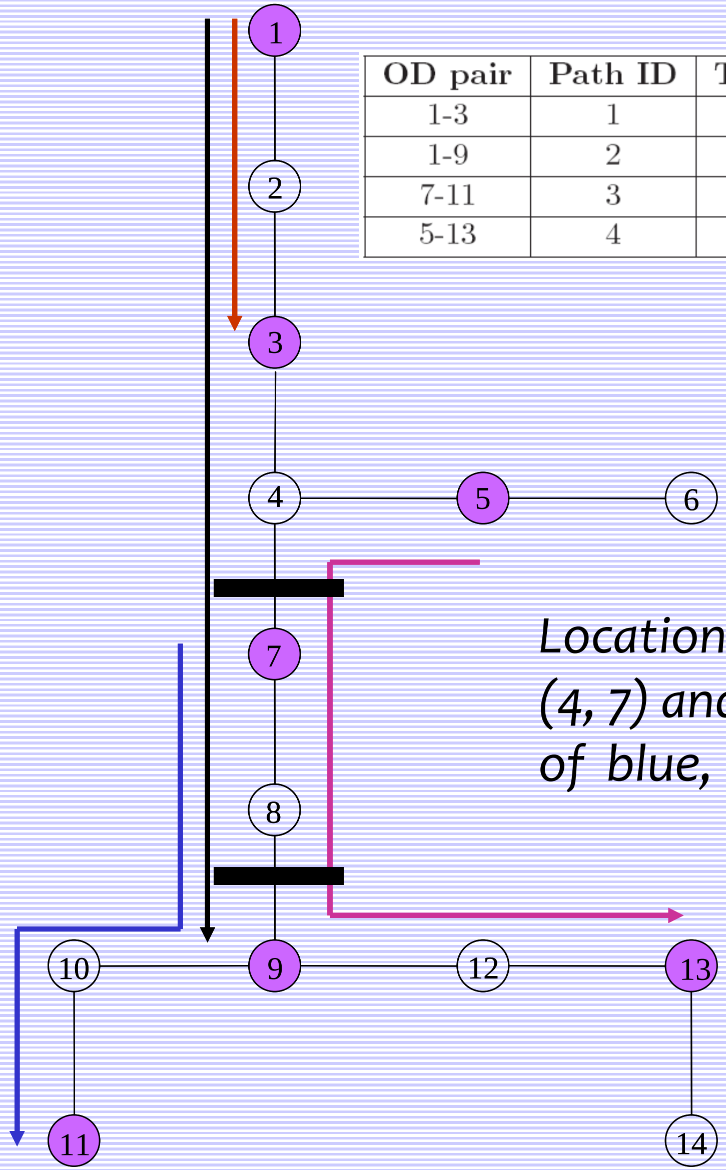


Location of AVI readers:
(1, 2) and (2,3) ⇒ travel time for the entire red path

Locate AVI Readers to Monitor Travel Times



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5-13	4	10	(5,4)	(4,7)	(7,8)	(8,9)	(9,12)	(12,13)

Location of AVI readers:
(4, 7) and (8,9) $\Rightarrow$ travel time on portions of blue, black and purple paths

Locate AVI Readers to Monitor Travel Times

Sensor Technologies and Sensors Measurements

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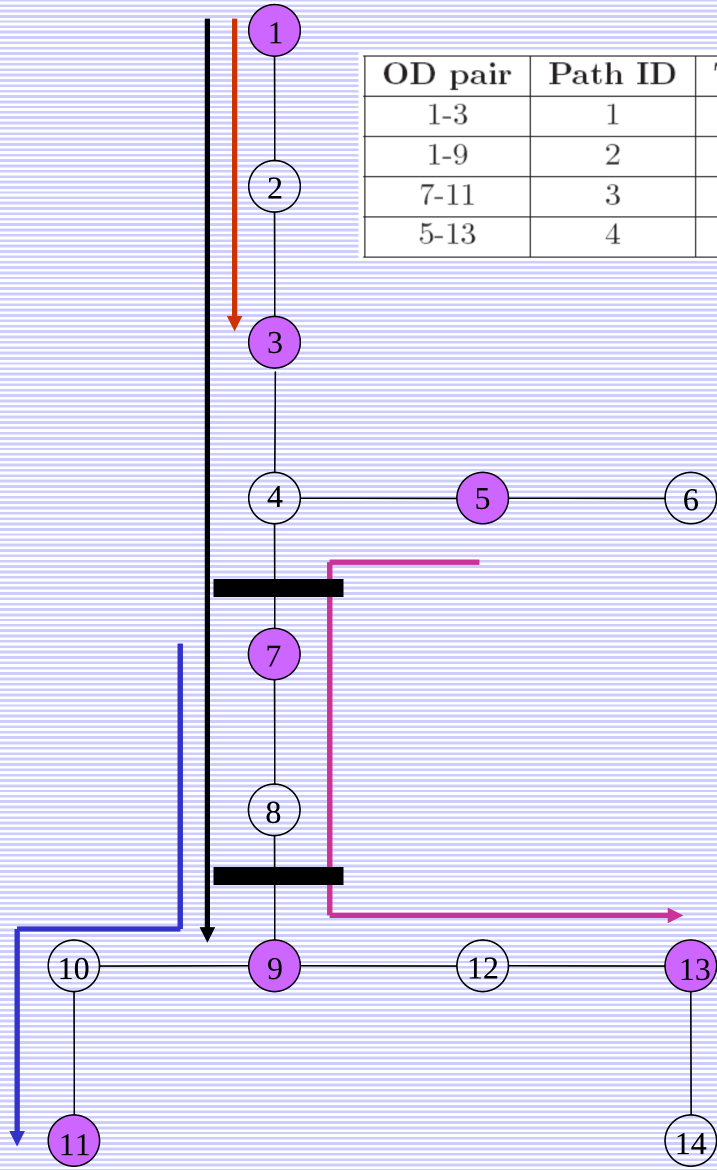

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Criteria for the location:

- *Number of readings*
- *Number of AVI readers*
- *Length of the readings*
- *Number of Covered OD pairs*
- *Flow captured*

Locate AVI Readers to Monitor Travel Times: Proposed Mathematical Models

Sensor Technologies and Sensors Measurements

Location of AVI Readers

Model AVI3

Model AVI4

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Model AVI4 + Sensor Failure

Conclusions

Model AVI1 :

locates B readers to maximize both the total number of readings and the total number of covered OD pairs

Model AVI2 :

associate different benefits to the OD pairs and maximize the total benefit accrued by locating at most B readers

Model AVI3 :

locate B readers to maximize total vehicle miles that are measured

Model AVI4:

locates B readers to maximize the reliability of the obtained travel time predictions

Locate AVI Readers to Monitor Travel Times: Model AVI3 – a simple case

Sensor Technologies and Sensors Measurements

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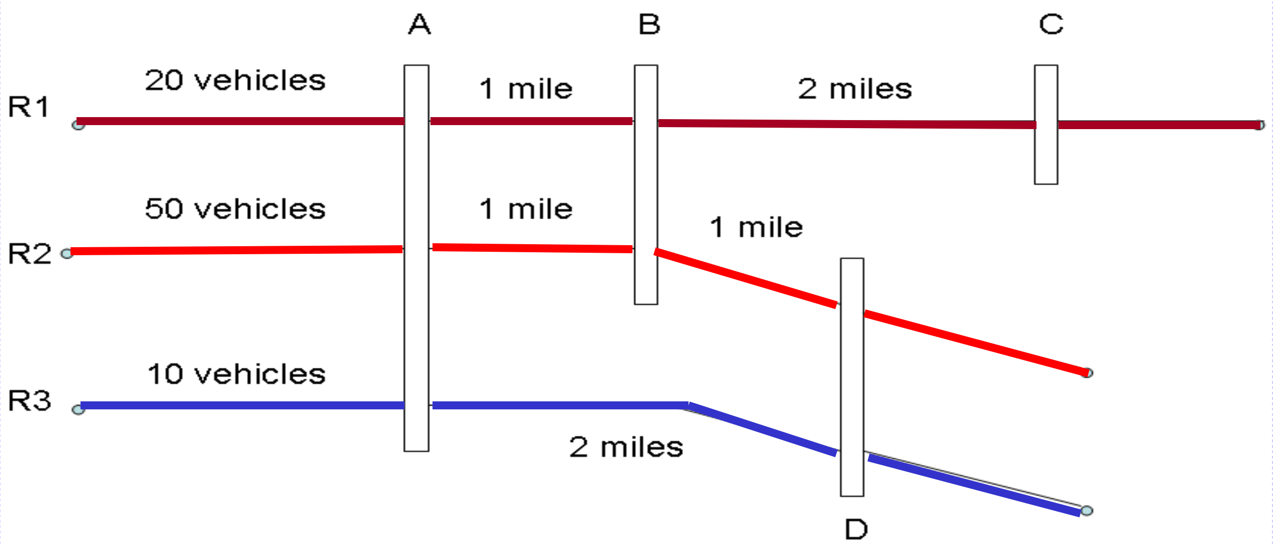


Three routes in the network:

R1 with a volume of 20 vehicles that pass locations A, B, and C

R2 with a volume of 50 vehicles that pass locations A, B, and D

R3 with a volume of 10 vehicles that pass locations A and D

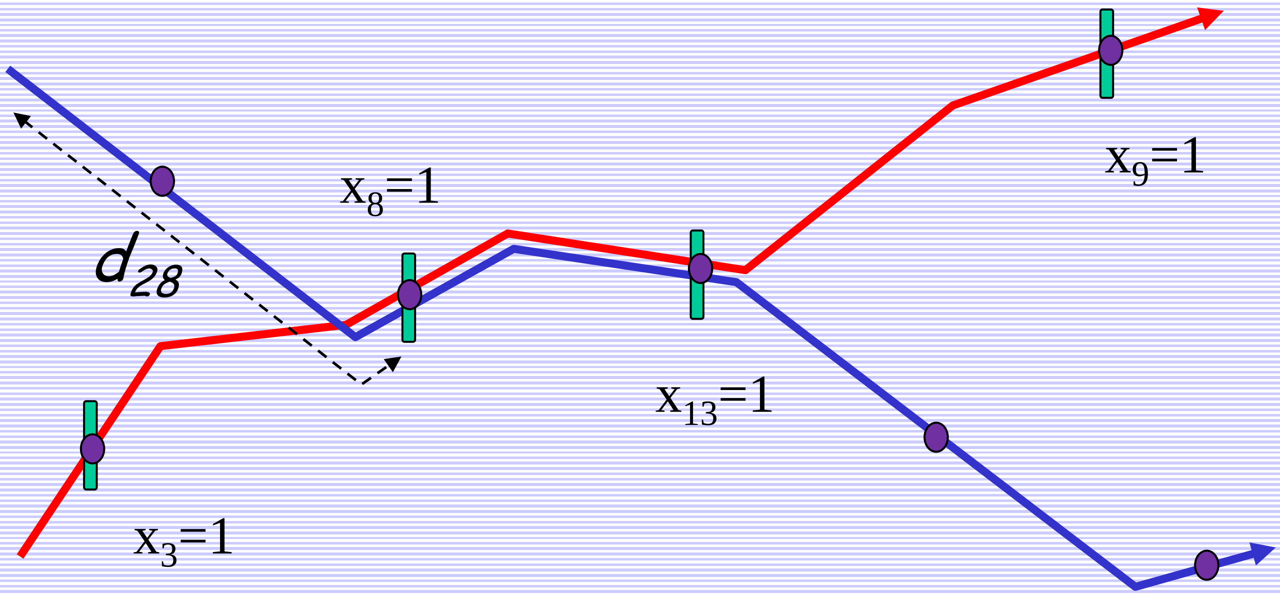
If two sensors are to be located, by locating them at A and D, we will measure *(50 + 10) vehicles x 2 miles = 120 vehicle miles*, which is the optimal solution.

Challenge!

Suppose we locate 4 ID sensors

Possible sites for sensors

Use binary decision variable x_j



x_8, x_{13} measure blue route (route 2)

and x_3, x_9 as well as (x_8, x_{13}) measure red route (route 1)

Challenge!

- Sensor Technologies and Sensors Measurements
- Location of AVI Readers
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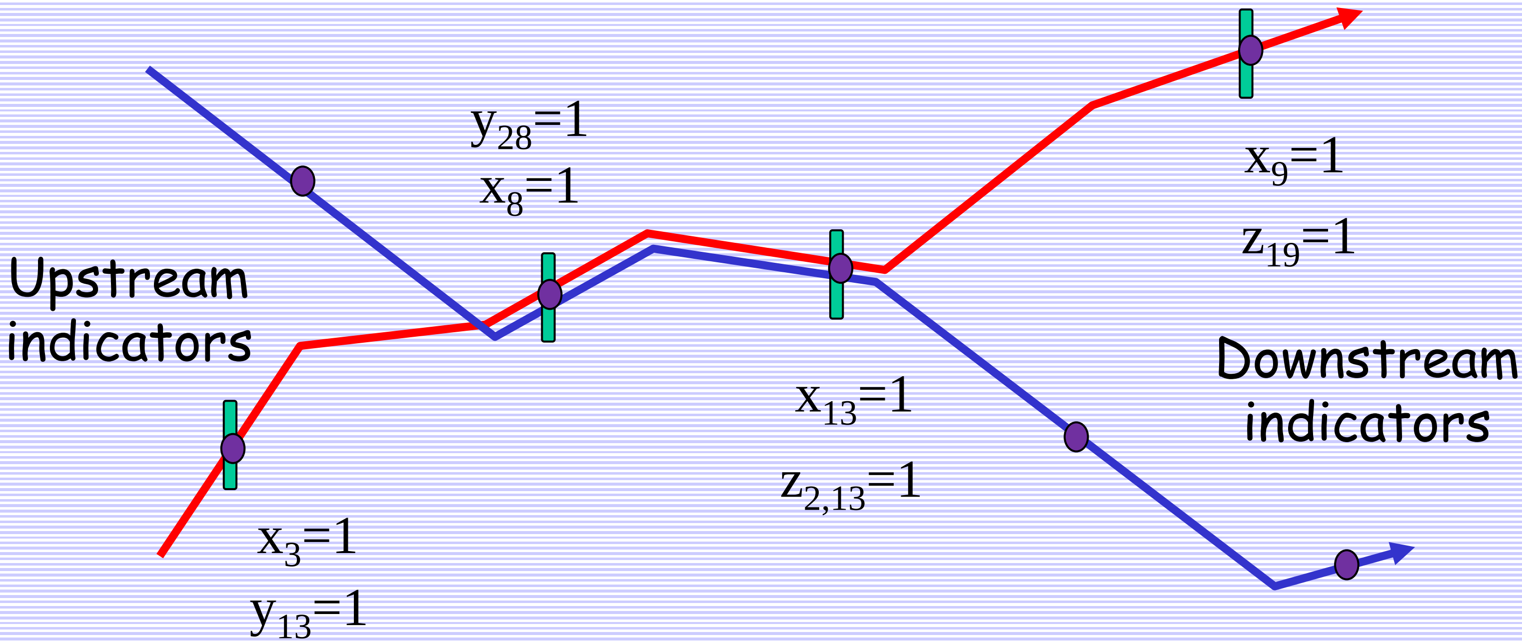
Suppose we locate 4 ID sensors

Used location decision variable x_j



Now use upstream/downstream decision variables y_{ij}, z_{ij}

Locate AVI Reader to Monitor Travel Times: Model AVI3 – formulation (1/4)

Indices:

- $i = 1, 2, \ldots, I$ represents the **set of routes**
- $j = 1, 2, \ldots, J$ represents the **set of potential sites** for locating sensors in the network

Parameters:

- d_{ij} is Site *j*'s distance from the origin of Route *i*
- v_i is the traffic volume on Route *i*
- *B* is the number of new sensors that can be installed (budget limits)
- *w* is the number of existing sensors in the network

Locate AVI Reader to Monitor Travel Times: Model AVI3 – formulation (2/4)

$$x_j = \begin{cases} 1 & \text{if a sensor is located at Site } j \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ij} = \begin{cases} 1 & \text{if Site } j \text{ is the most downstream site in Route } i \text{ with a sensor installed} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if Site } j \text{ is the most upstream site in Route } i \text{ with a sensor installed} \\ 0 & \text{otherwise} \end{cases}$$

Locate AVI Reader to Monitor Travel Times: Model AVI3 – formulation (3/4)

Sensor Technologies and Sensors Measurements

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Conclusions

Maximize $$\sum_{i=1}^{I}\sum_{j\in R_i}(z_{ij}-y_{ij})d_{ij}v_i$$

$$\sum_{j=1}^{J}x_j \le B+w$$

$x_j = 1 \quad \forall j$ where there exists a sensor already

$$y_{ij} \le x_j \quad \forall i \quad \forall j$$

$$z_{ij} \le x_j \quad \forall i \quad \forall j$$

$$\sum_{j\in R_i} y_{ij} \le 1 \quad \forall i$$

$$\sum_{j\in R_i} z_{ij} \le 1 \quad \forall i$$

$$\sum_{j\in R_i} y_{ij} - \sum_{j\in R_i} z_{ij} = 0 \quad \forall i$$

Locate sensors on different routes in the network so that the total vehicle miles measured on the routes is maximized

Locate AVI Reader to Monitor Travel Times: Model AVI3 – formulation (4/4)

Maximize $\sum_{i=1}^{I} \sum_{j \in R_i} (z_{ij} - y_{ij}) d_{ij} v_i$

$$\sum_{j=1}^{J} x_j \le B + w$$

Locate *B* sensors

$x_j = 1 \quad \forall j$ where there exists a sensor already

$$y_{ij} \le x_j \quad \forall i \quad \forall j$$

$$z_{ij} \le x_j \quad \forall i \quad \forall j$$

A Site cannot be either the most upstream or downstream checkpoint without a sensor installed

$$\sum_{j \in R_i} y_{ij} \le 1 \quad \forall i$$

$$\sum_{j \in R_i} z_{ij} \le 1 \quad \forall i$$

A route has at most one most upstream and downstream checkpoints respectively

$$\sum_{j \in R_i} y_{ij} - \sum_{j \in R_i} z_{ij} = 0 \quad \forall i$$

If there is a most upstream checkpoint in a route, there must exist a most downstream one and vice versa

Locate AVI Reader to Monitor Travel Times: Proposed Mathematical Models

Sensor Technologies and Sensors Measurements

Location of AVI Readers

Model AVI3

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Model AVI4:

locates B readers to maximize the reliability of the obtained travel time predictions

The motivation for this objective function is for traffic managers to inform to commuters as reliable as possible estimates for travel times for route segments.

Locate AVI Reader to Monitor Travel Times: Model AVI4 – concept & findings (1/3)

Sensor Technologies and Sensors Measurements

Location of AVI Readers

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Model AVI4 + Sensor Failure

Conclusions

We want to improve our ability to predict network travel times by sampling vehicle travel times with a prescribed number of sensors installed in the network

Due to the dynamic and stochastic nature of the transportation network, the prediction of travel times is represented by confidence intervals for different time periods

We need to determine on which of the routes to locate the sensors to minimize the variance of the predictive distribution of network travel times in order to improve confidence intervals

Locate AVI Reader to Monitor Travel Times: Model AVI4 – concept & findings (2/3)

- Sensor Technologies and Sensors Measurements
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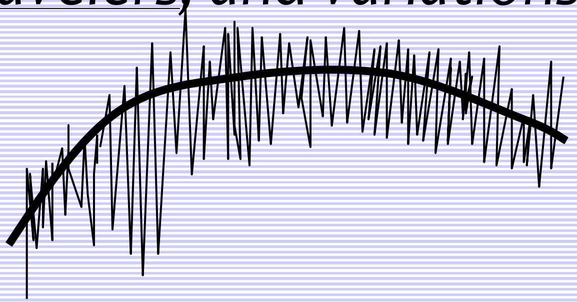
Travel time on an arc is a random variable, that includes variations due to travelers, and variations due to traffic conditions.



At any time, component dues to traffic conditions, has mean μ_i, which remains constant over short periods of time; μ_i has a mean η_i and variance γ_i (assume Normal and known from data)

With the observed travel times, by appealing to the Bayesian statistical analysis (prior → posterior), the mean travel time distribution on the segment is updated

Locate AVI Reader to Monitor Travel Times: Model AVI4 – concept & findings (3/3)

Sensor Technologies and Sensors Measurements

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Conclusions

For any pair of routes R_1 and R_2 with volumes V_1 and V_2 and prior knowledge of the means and variances of μ_i on both routes, the following sampling rule can be proven:

Sample the route R with higher value of

f(R) = volume + β(R) x prior travel-time variance

where β(R) route-specific parameter,

to get a higher decrease in the variance of the predicted mean network wide travel time.

The general network sensor location problem that minimizes the posterior variance of network travel time distribution can be formulated as an integer program based on the above rule (due to Bayesian statistics), with constraints similar to AVI_3

Formulation for Model AVI4

Sensor Technologies and Sensors Measurements

Location of AVI Readers

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Model AVI4 + Sensor Failure

Conclusions

$$\max \frac{1}{2} \sum_{i=1}^{I} \sum_{j \in R_i} \sum_{k \in R_i} \left(y_{ij} + z_{ik} \right) \Delta \gamma_i^{jk}$$

Objective is the max. sum of reductions in variance of the mean travel time on Route *i* if travel times can be measured from Site *j* to Site *k* on that route (i.e. for segment *d*: $\gamma_d - \gamma_d{}^*$)

$\Rightarrow$ Locate sensors on different routes in the network thus that the total variance reduction of travel time prediction can be maximized

Subject to: other constraints similar as Model AVI3.

For those interested, objective of Model AVI4

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$$\Delta\gamma_i^{jk} = \gamma_d - \gamma_d^*$$

Indices *i*,*j*,*k* define the segment *d* from Site j to Site *k* on Route *i*

Prior

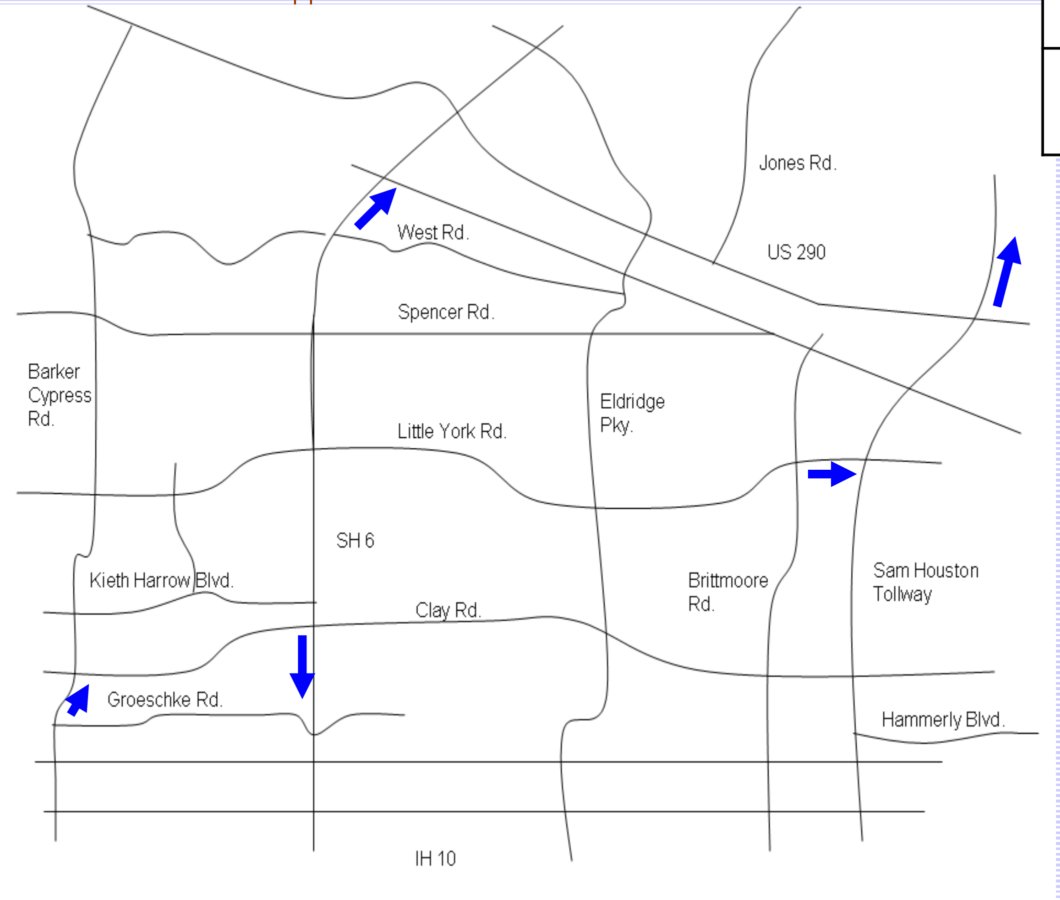
$$\gamma_d = \sum_{m\in d}\gamma_m$$

Posterior

$$\gamma_d^* = \frac{\sum_{m\in d}\gamma_m \sum_{m\in d}\sigma_m^2}{n_d\sum_{m\in d}\gamma_m + \sum_{m\in d}\sigma_m^2}$$

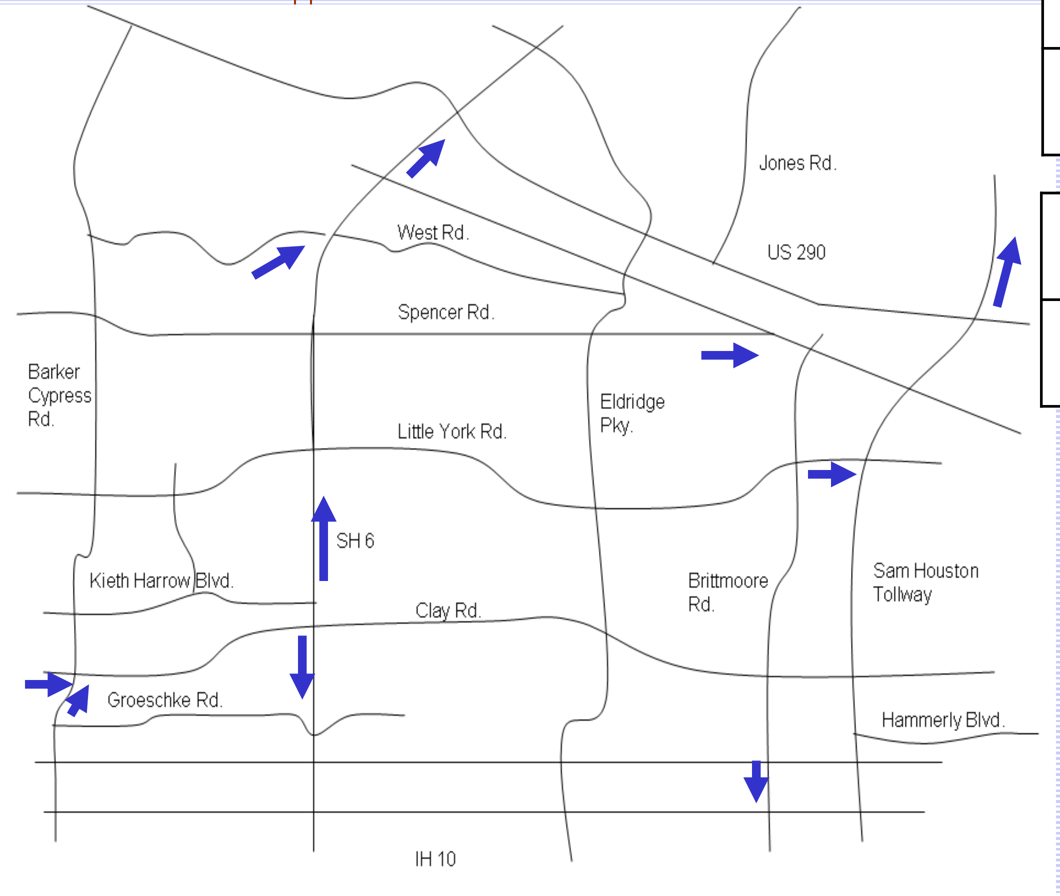
Study Area Network

Install *B* = 5 Sensors



B = 5	Variance reduced	Percentage of total	Increase by
	7,578.79 (3,361.08)	60.25% (26.72%)	33.53%

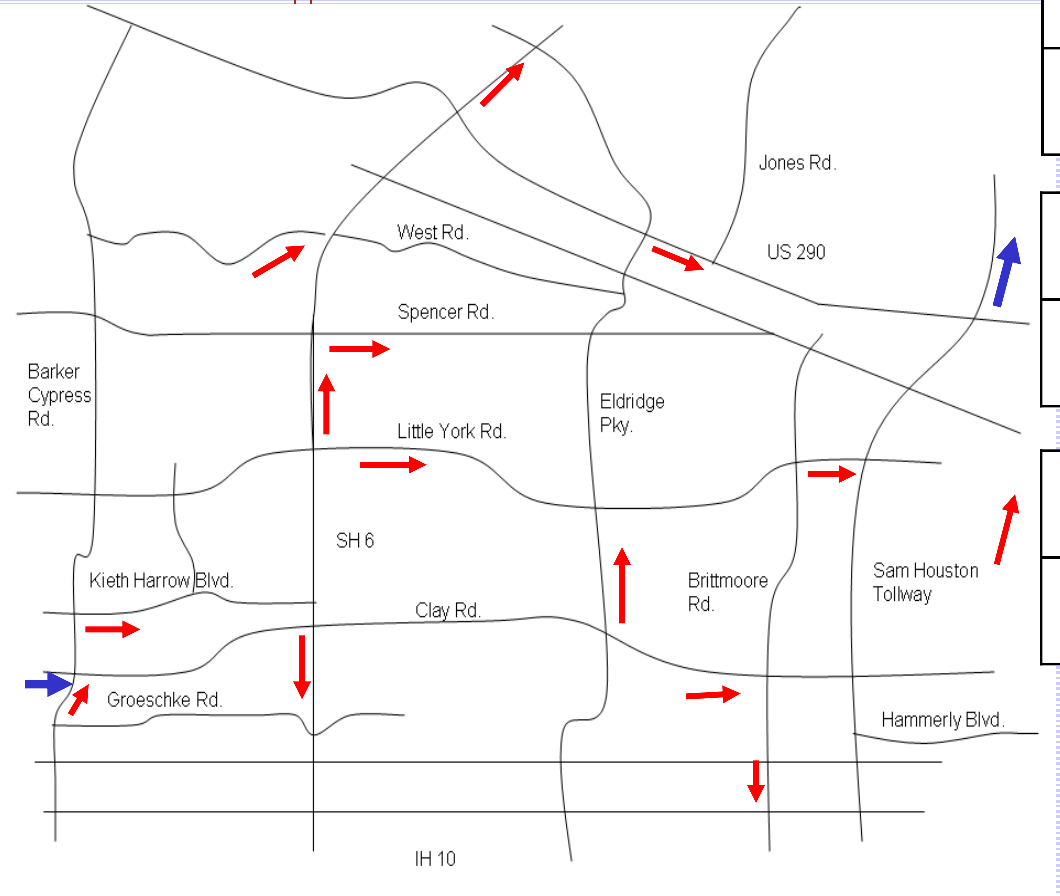
Install B = 10 Sensors



B = 5	Variance reduced	Percentage of total	Increase by
	7,578.79 (3,361.08)	60.25% (26.72%)	33.53%

B = 10	Variance reduced	Percentage of total	Increase by
	8,997.69	71.53%	11.28%

Install *B* = 15 Sensors



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B = 10	Variance reduced	Percentage of total	Increase by
	8,997.69	71.53%	11.28%

B = 15	Variance reduced	Percentage of total	Increase by
	9,908.40	78.77%	7.24%

Extensions of models (1/2)

Link failure:
Some links in the network have a known probability of disruption due to external events.

The model seeks for the optimal location of B sensors to minimize the expected posterior variance over all possible link failure scenarios.

Sensor failure:
We assume each sensor has a failure probability of q.

The resulting model minimizes the expected posterior variance in the prediction of travel time when B sensors are to be located.

Extensions of models (2/2)

Sensor Technologies and Sensors Measurements

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Conclusions

Model AVI3 + Sensor Failure :

Addressed by Li and Ouyang (2010)

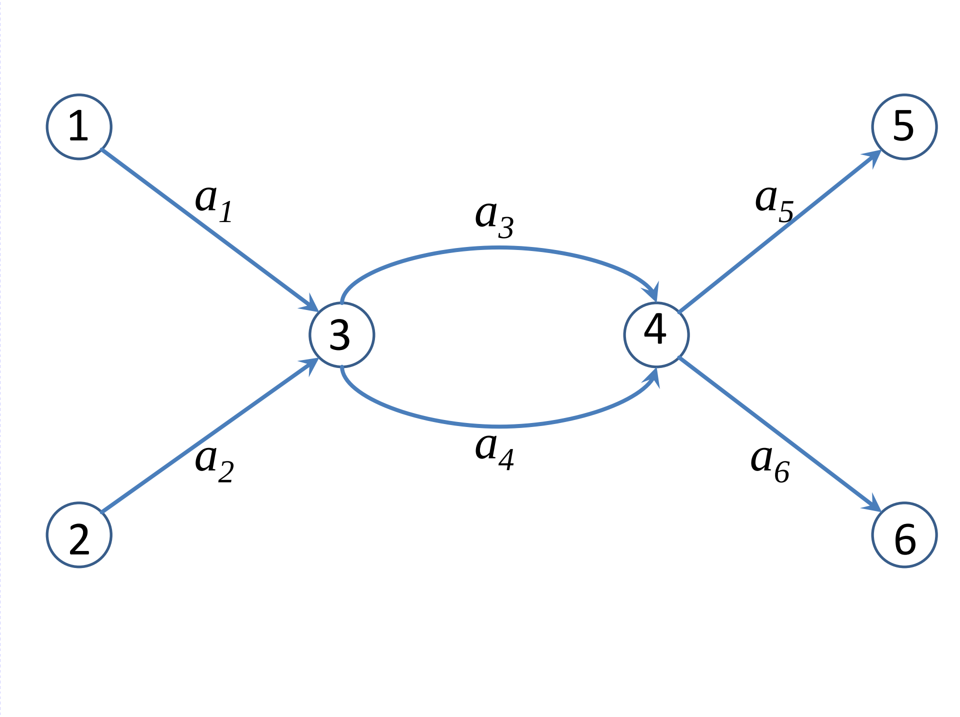
We consider:

Model AVI3 + Link Failure

Model AVI4 + Link Failure

Model AVI4 + Sensor Failure

Model AVI4 + Link failure



Route $R_1 = \{a_1, a_3, a_5\}$

Route $R_2 = \{a_1, a_4, a_5\}$

Route $R_3 = \{a_2, a_3, a_5\}$

Route $R_4 = \{a_2, a_4, a_5\}$

Route $R_5 = \{a_1, a_3, a_6\}$

Route $R_6 = \{a_1, a_4, a_6\}$

Route $R_7 = \{a_2, a_3, a_6\}$

Route $R_8 = \{a_2, a_4, a_6\}$

Model AVI4 + Link failure

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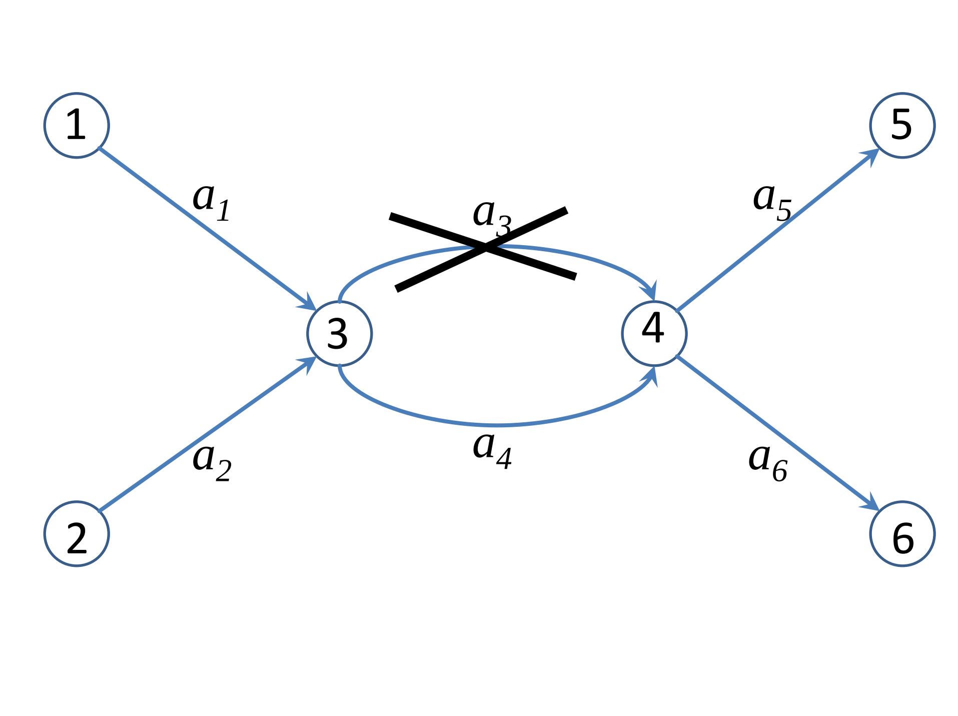

If link a_3 fails then **routes and flows** on the network change.

Each link failure corresponds to a possible **network scenario**

~~Route $R_1 = \{a_1, a_3, a_5\}$~~
Route $R_2 = \{a_1, a_4, a_5\}$
~~Route $R_3 = \{a_2, a_3, a_5\}$~~
Route $R_4 = \{a_2, a_4, a_5\}$

~~Route $R_5 = \{a_1, a_3, a_6\}$~~
Route $R_6 = \{a_1, a_4, a_6\}$
Route R_7 ~~$= \{a_2, a_3, a_6\}$~~
Route $R_8 = \{a_2, a_4, a_6\}$

Model AVI4 + Link failure

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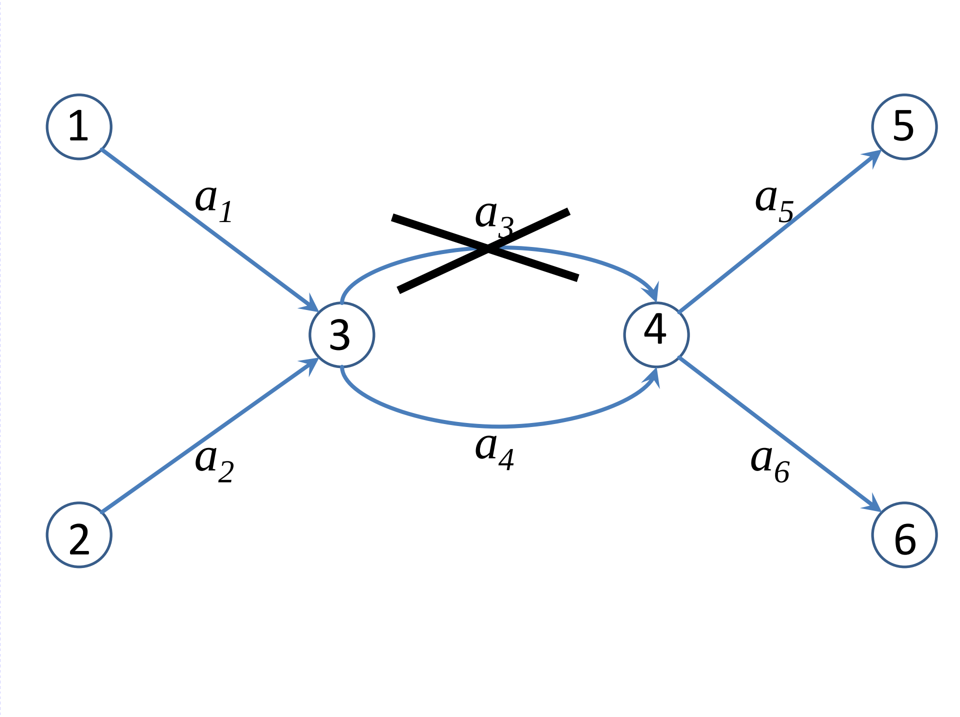

If routes on the network change then:

- flows on the routes change
- values $\Delta\gamma_i^{jk}$ change

Model AVI4 + Link failure: Mathematical Formulation

Sensor Technologies and Sensors Measurements

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Conclusions

Indices:

- $i = 1, 2, \ldots, I$ represents the **set of routes**
- j or $k = 1, 2, \ldots, J$ represents the **set of potential sites** for locating sensors in the network
- s= 1,2,… S represents the **set of possible scenarios** for link failure

Parameters:

- $\Delta\gamma_{is}^{jk}$ decrement in variance of mean route travel times when Route i is observed from Site j *to* Site k during the scenario s
- p_s is the probability for scenario s
- B is the number sensors that can be installed (budget limits)

Model AVI4 + Link failure: Mathematical Formulation

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Conclusions

$$x_j = \begin{cases} 1 & \text{if a sensor is located at Site } j \\ 0 & \text{otherwise} \end{cases}$$

$$z_{iks} = \begin{cases} 1 & \text{if Site } k \text{ is the most downstream site in Route } i \text{ with a sensor installed} \\ & \text{on scenario } s \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ijs} = \begin{cases} 1 & \text{if Site } j \text{ is the most upstream site in Route } i \text{ with a sensor installed} \\ & \text{on scenario } s \\ 0 & \text{otherwise} \end{cases}$$

Model AVI4 + Link failure: Mathematical Formulation

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$$\max \frac{1}{2} \sum_{s=1}^{S} \sum_{i=1}^{I} \sum_{j \in R_i} \sum_{k \in R_i} \Delta\gamma_{is}^{jk} (y_{ijs} + z_{iks}) p_s$$

$$\sum_{j=1}^{J} x_j \leq B$$

$$y_{ijs} \leq x_j \qquad \forall j \quad \forall i \quad \forall s$$

$$z_{iks} \leq x_j \qquad \forall k \quad \forall i \quad \forall s$$

Locate sensors on different routes in the network so that the expected variance travel time reduction on the routes is maximized

Model AVI4 + Link failure: Mathematical Formulation

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$$\max \frac{1}{2} \sum_{s=1}^{S} \sum_{i=1}^{I} \sum_{j \in R_i} \sum_{k \in R_i} \Delta\gamma_{is}^{jk} (y_{ijs} + z_{iks}) p_s$$

$$\sum_{j=1}^{J} x_j \leq B$$

Locate *B* sensors

$$y_{ijs} \leq x_j \quad \forall j \quad \forall i \quad \forall s$$

$$z_{iks} \leq x_j \quad \forall k \quad \forall i \quad \forall s$$

A site cannot be either the most upstream or downstream checkpoint without a sensor installed

Model AVI4 + Link failure: Mathematical Formulation

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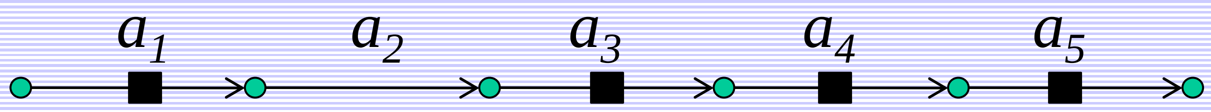
$$\sum_{j \in R_i} y_{ijs} \leq 1 \qquad \forall i \quad \forall s$$

$$\sum_{k \in R_i} z_{iks} \leq 1 \qquad \forall i \quad \forall s$$

$$\sum_{j \in R_i} y_{ijs} - \sum_{j \in R_i} z_{ijs} = 0 \qquad \forall i \quad \forall s$$

If there is a most upstream checkpoint in a route, there must exist a most downstream one and vice versa

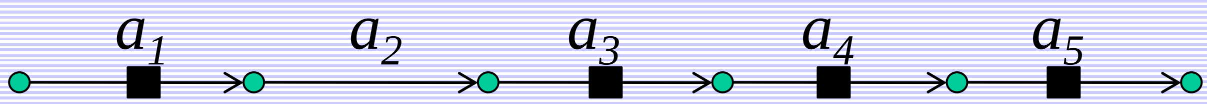
Model AVI4 + Sensor failure



Assume probability failure for each sensor is equal to q

If we consider all the possible failure cases, our objective function becomes the maximization of the *expected total reduction in the variance* of mean route travel times

Model AVI4 + Sensor failure



Let us consider the random variable

S=number of failures

$Pr(S=0) = q^0 (1-q)^4$

$Pr(S=1) = q^1 (1-q)^3$

$Pr(S=2) = q^2 (1-q)^2$

$Pr(S=3) = q^3 (1-q)^1$

$Pr(S=4) = q^4 (1-q)^0$

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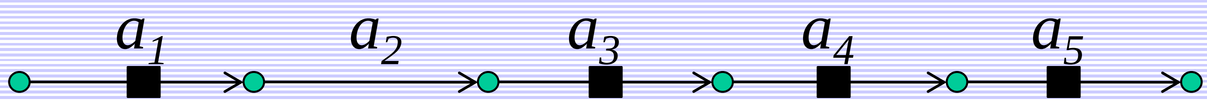

Let us evaluate the expected value of our function:

The event S=0 corresponds to a value equal to $\Delta\gamma^{a_1 a_5}$

The event S=1 corresponds to a value equal to:

-if a_1 fails (we evaluate the subroute a_3-a_5) → $\Delta\gamma^{a_3 a_5}$

-if a_3 fails (we evaluate the subroute a_1-a_5) → $\Delta\gamma^{a_1 a_5}$

-if a_4 fails (we evaluate the subroute a_1-a_5) → $\Delta\gamma^{a_1 a_5}$

-if a_5 fails (we evaluate the subroute a_1–a_4) → $\Delta\gamma^{a_1 a_4}$

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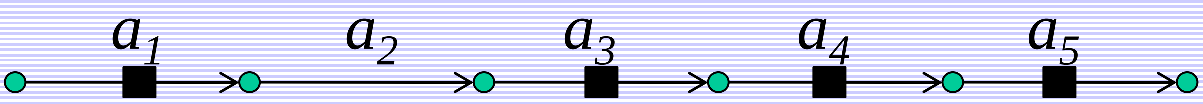

S=2 corresponds to a value equal to:

-if a_1 and a_3 fail (we evaluate the subroute a_4-a_5)→ $\Delta\gamma^{a_4 a_5}$

-if a_1 and a_4 fail (we evaluate the subroute a_3-a_5)→ $\Delta\gamma^{a_3 a_5}$

-if a_1 and a_5 fail (we evaluate the subroute a_3-a_4) → $\Delta\gamma^{a_3 a_4}$

-if a_3 and a_4 fail (we evaluate the subroute a_1-a_5) → $\Delta\gamma^{a_1 a_5}$

-if a_3 and a_5 fail (we evaluate the subroute a_1-a_4)→ $\Delta\gamma^{a_1 a_4}$

-if a_4 and a_5 fail (we evaluate the subroute a_1-a_3) → $\Delta\gamma^{a_1 a_3}$

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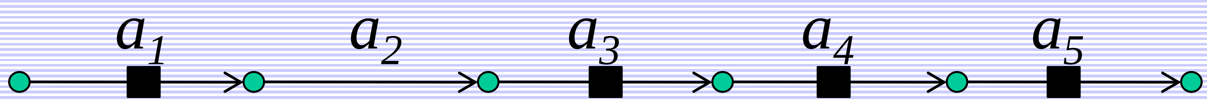

The event S=3 corresponds to a value equal to 0 for each triple of sensors that fail

The event S=4 corresponds to a value equal to 0

The expected value of variance reduction is then:

$$\Delta\gamma^{a_1a_5}(1-q)^4 +$$

$$(\Delta\gamma^{a_3a_5} + \Delta\gamma^{a_1a_5} + \Delta\gamma^{a_1a_5} + \Delta\gamma^{a_1a_4})q(1-q)^3 +$$

$$(\Delta\gamma^{a_4a_5} + \Delta\gamma^{a_3a_5} + \Delta\gamma^{a_3a_4} + \Delta\gamma^{a_1a_5} + \Delta\gamma^{a_1a_4} + \Delta\gamma^{a_1a_3})q^2(1-q)^2$$

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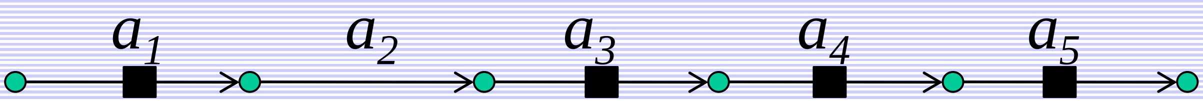

We could compute the expected value also in a different way by observing that it depends only on the active upstream and downstream sensors.

Sensor on a_1 is the 1st active upstream sensor
(we say it is active at ***level 0***)

Sensor on a_3 is the 2nd active upstream sensor
(we say it is active at ***level 1***)

Sensor on a_4 is the 3rd active upstream sensor
(we say it is active at ***level 2***)

Sensor on a_5 is the 4th active upstream sensor
(we say it is active at ***level 3***)

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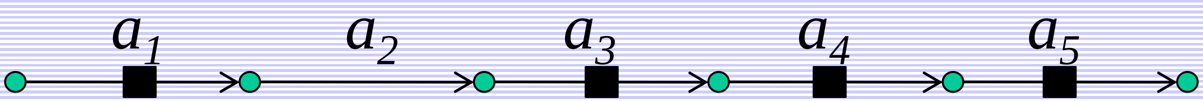

Sensor on a_5 is the 1st active downstream sensor (we say it is active at ***level 0***)

Sensor on a_4 is the 2nd active downstream sensor (we say it is active at ***level 1***)

Sensor on a_3 is the 3rd active downstream sensor (we say it is active at ***level 2***)

Sensor on a_1 is the 4th active downstream sensor (we say it is active at ***level 3***)

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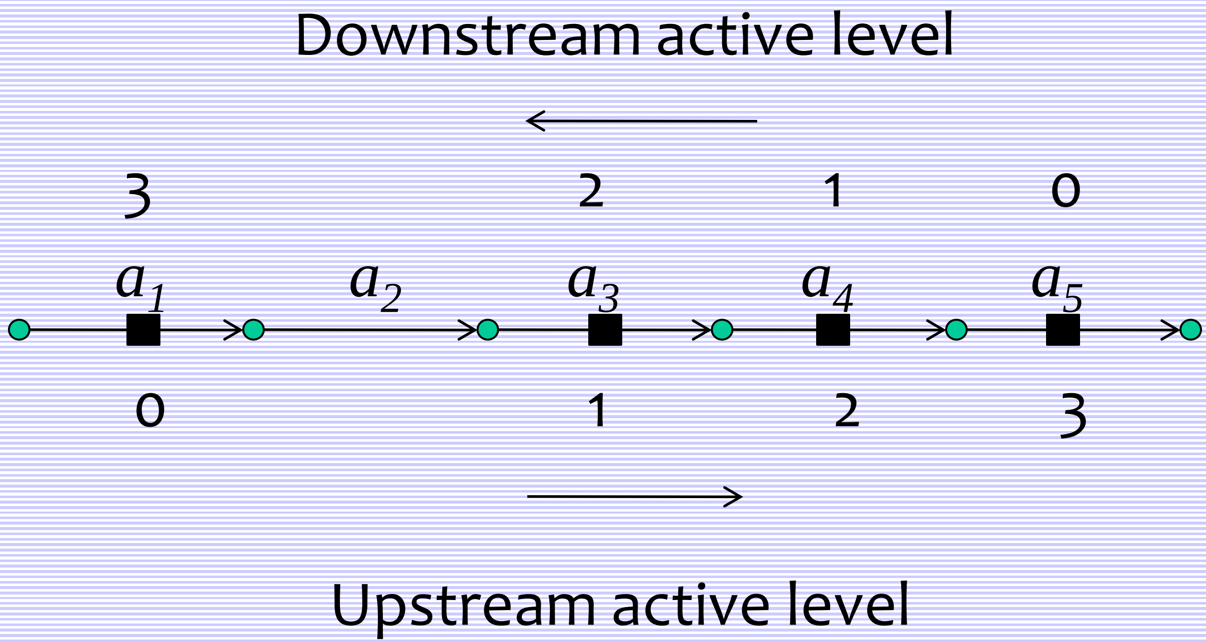
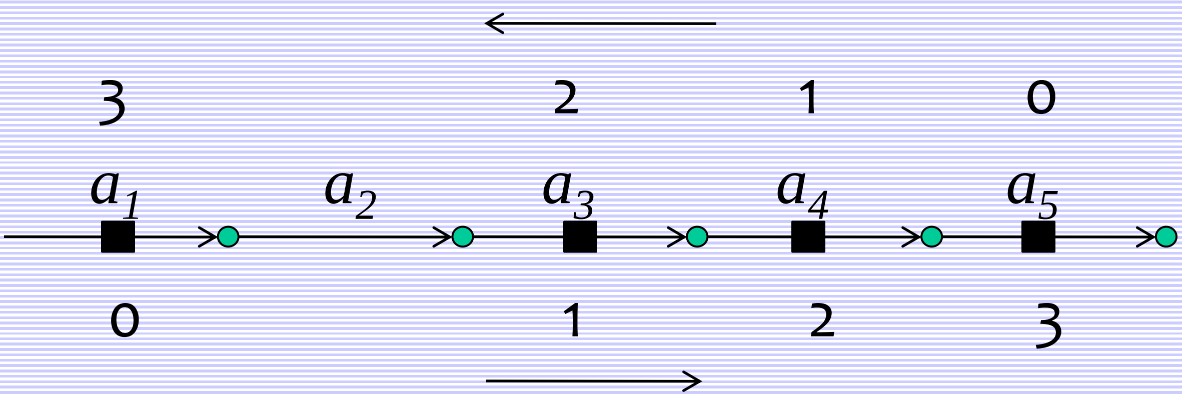


Model AVI4 + Sensor failure



The probability that a_1 is the **active upstream** sensor is (1-q)

The probability that a_3 is the **active upstream** sensor is q(1-q)

The probability that a_4 is the **active upstream** sensor is $q^2(1-q)$

The probability that a_5 is the **active upstream** sensor is $q^3(1-q)$

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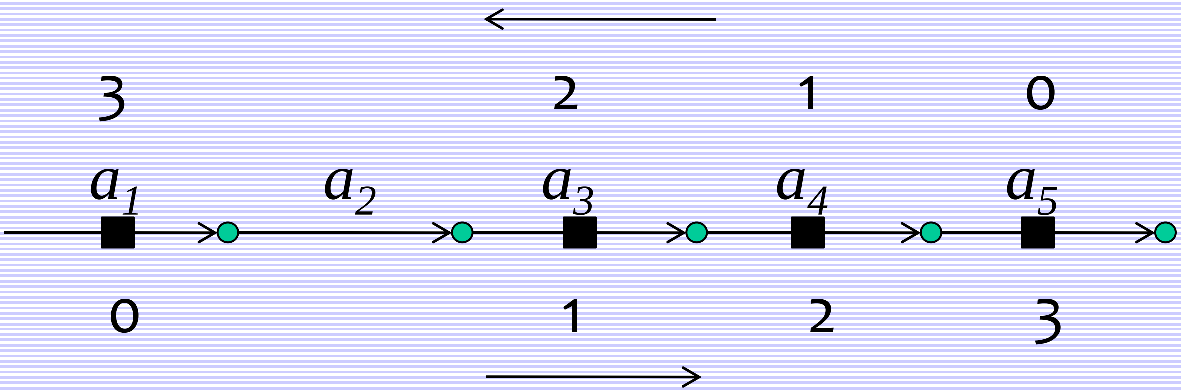

The probability that a_5 is the **active downstream** sensor is (1-q)

The probability that a_4 is the **active downstream** sensor is q(1-q)

The probability that a_3 is the **active downstream** sensor is $q^2(1-q)$

The probability that a_1 is the **active downstream** sensor is $q^3(1-q)$

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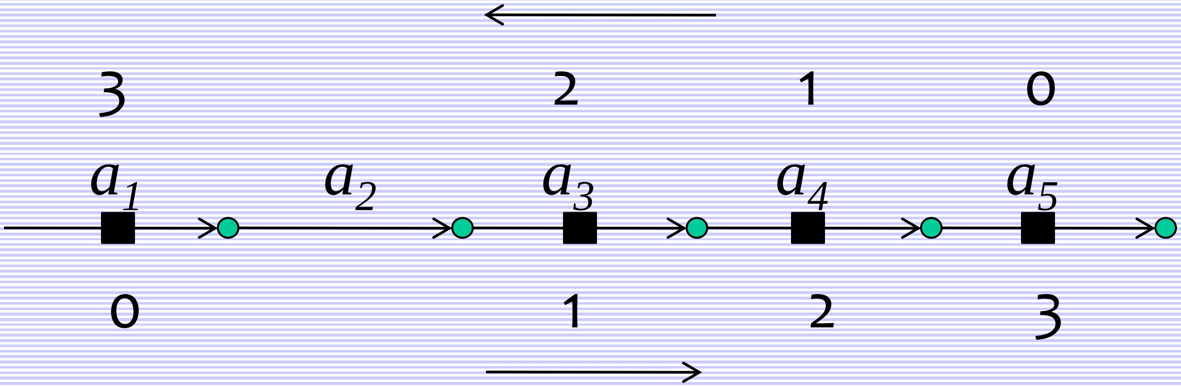
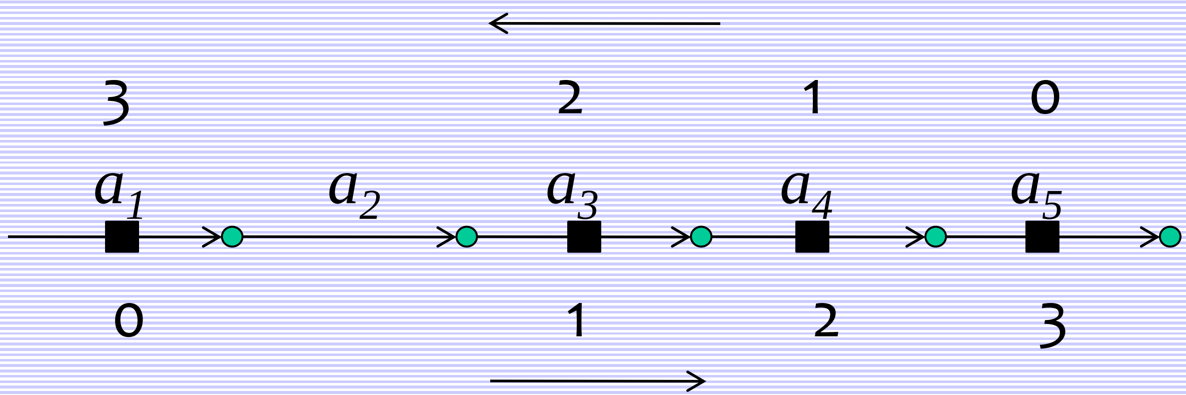


Considering the probability of each sensor to be the active upstream/downstream sensor we can evaluate the probability of monitoring each possible subroute

Model AVI4 + Sensor failure



The subroute a_1 -a_5 is monitored if a_1 is the active upstream and a_5 is the active downstream

→ a_1 is the active upstream at level 0 and a_5 is the active downstream at level 0

→ $(1-q)^2 q^0 q^0$

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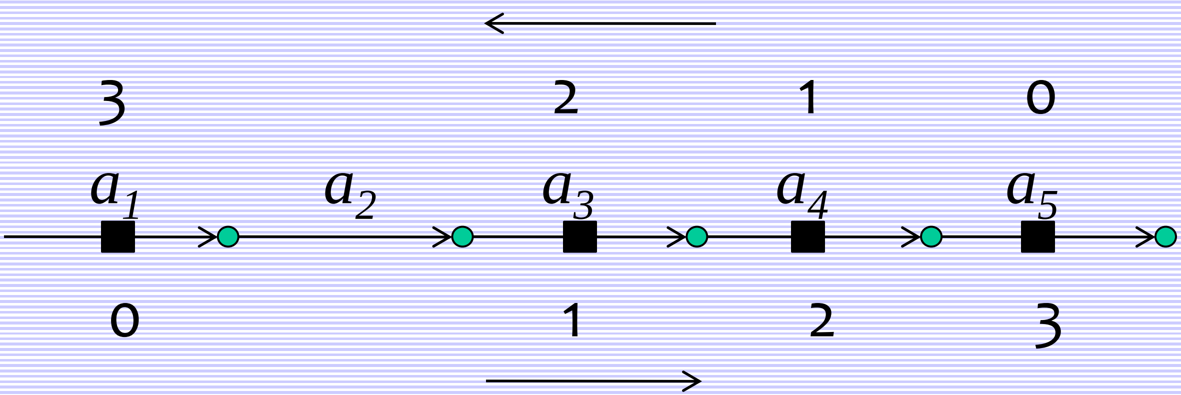

The subroute a_1 –a_4 is monitored if a_1 is the active upstream and a_4 is the active downstream

→ a_1 is the active upstream at level 0 and a_4 is the active downstream at level 1

→ $(1-q)^2 q^0 q^1$

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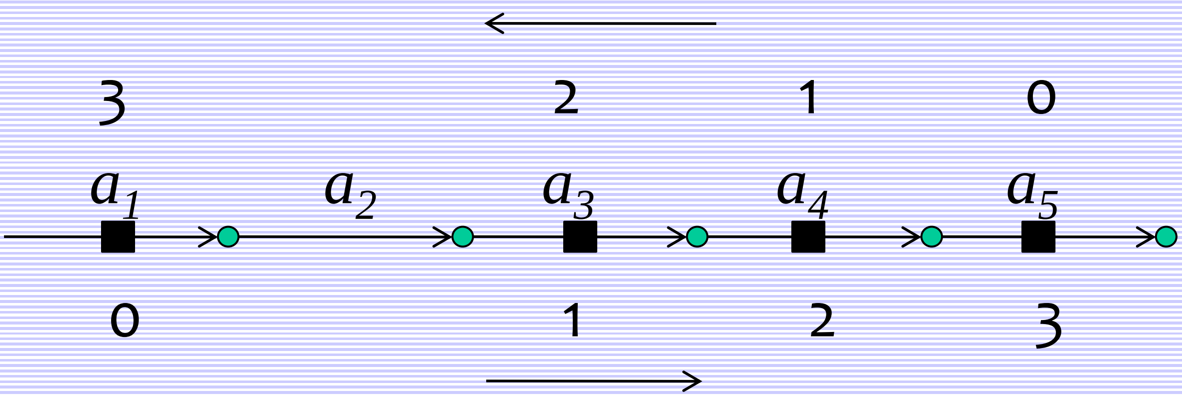

The subroute a_1 –a_3 is monitored if a_1 is the active upstream and a_3 is the active downstream

→ a_1 is the active upstream at level 0 and a_3 is the active downstream at level 2

→ $(1-q)^2 q^0 q^2$

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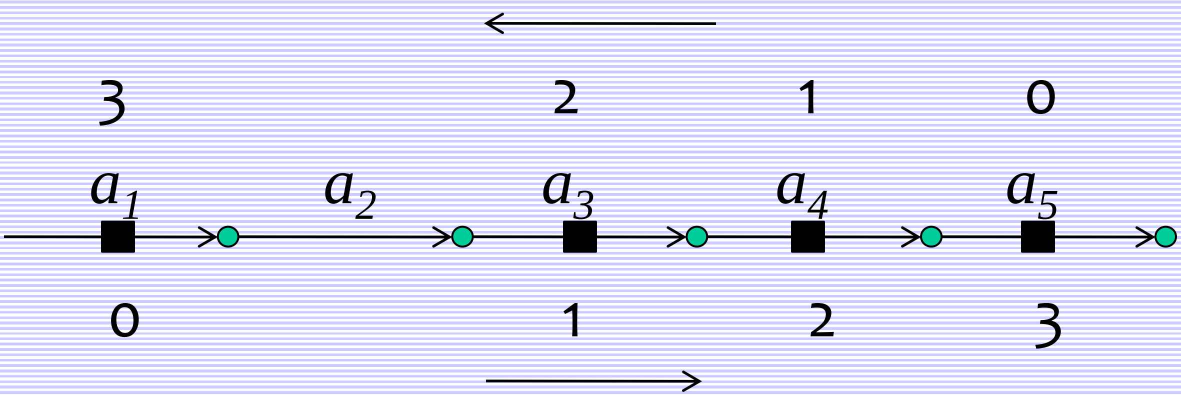

The subroute a_3–a_5 is monitored if a_3 is the active upstream and a_5 is the active downstream

→ a_3 is the active upstream at level 1 and a_5 is the active downstream at level 0

→ $(1\text{-}q)^2\, q^1\, q^0$

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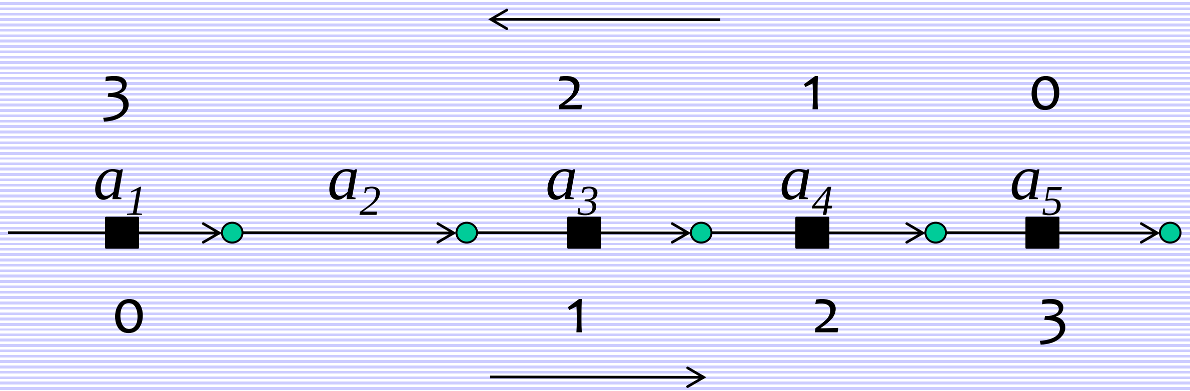

The subroute $a_3 - a_4$ is monitored if a_3 is the active upstream and a_4 is the active downstream

→ a_3 is the active upstream at level 1 and a_4 is the active downstream at level 1

→ $(1-q)^2 q^1 q^1$

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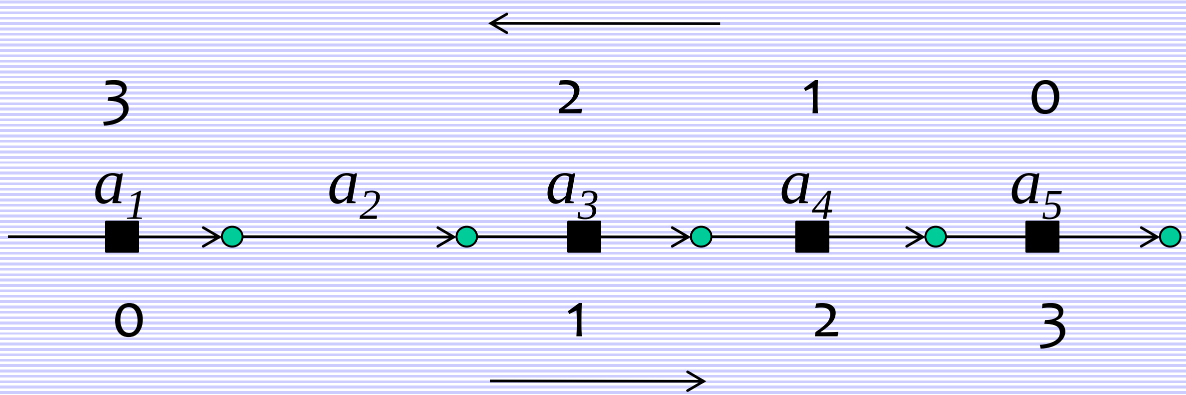

The subroute a_4–a_5 is monitored if a_4 is the active upstream and a_5 is the active downstream

→ a_4 is the active upstream at level 2 and a_5 is the active downstream at level 0

→ $(1-q)^2\, q^2\, q^0$

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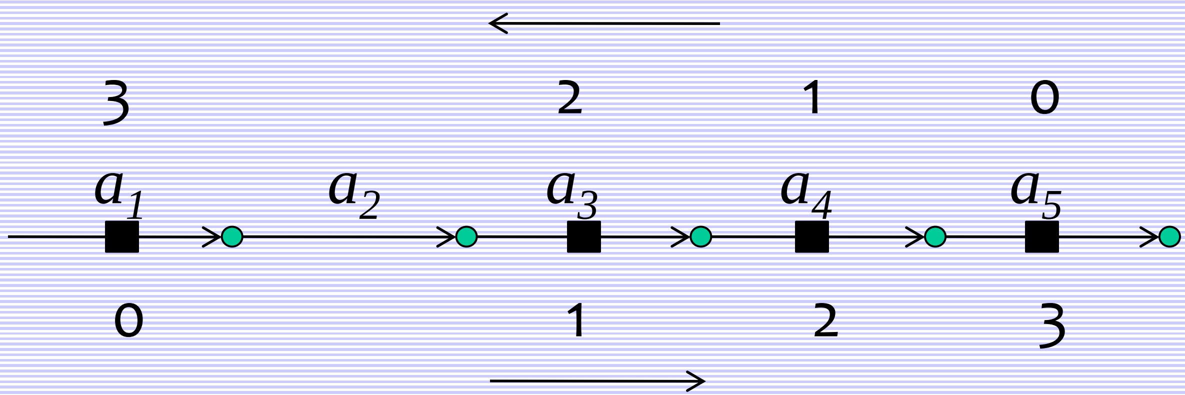

In general, the probability to monitor the subroute a_j–a_k if a_j is an active upstream al level h and a_k is an active downstream at level t is

$$(1-q)^2 q^{h+t}$$

Model AVI4 + Sensor failure: Mathematical Formulation

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Indices:

- $i = 1, 2, \ldots, I$ represents the **set of routes**
- j or $k = 1, 2, \ldots, J$ represents the **set of potential sites** for locating sensors in the network
- h or $t = 0, 1, 2, \ldots |R_i|$ represents the **set of possible levels** for sensors (upstream/downstream) on route R_i

Parameters:

- $\Delta\gamma_i^{jk}$ reduction in variance of mean route travel times when route R_i is observed from Site j to Site k
- q is the failure probability for each located sensor
- B is the number of sensors that can be installed (budget limits)

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$$x_j = \begin{cases} 1 & \text{if a sensor is located at Site } j \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ikt} = \begin{cases} 1 & \text{if Site } k \text{ is the most downstream site of level } t \text{ in Route } i \text{ with a sensor installed} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ijh} = \begin{cases} 1 & \text{if Site } j \text{ is the most upstream site of level } h \text{ in Route } i \text{ with a sensor installed} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{iht}^{jk} = \begin{cases} 1 & \text{if Site } j \text{ is the most upstream site of level } h \text{ in Route } i \text{ with a sensor installed and} \\ & \text{Site } k \text{ is the most downstream site of level } t \text{ in Route } i \text{ with a sensor installed} \\ 0 & \text{otherwise} \end{cases}$$

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$$\max \sum_{i=1}^{I} \sum_{j \in R_i} \sum_{k \in R_i} \sum_{h=0}^{|R_i|} \sum_{t=0}^{|R_i|} \Delta\gamma_i^{jk} \delta_{iht}^{jk} (1-q)^2 q^{h+t}$$

$$\sum_{j=1}^{J} x_j \le B$$

$$\sum_{h=0}^{|R_i|} y_{ijh} = x_j \qquad \forall j \quad \forall i$$

$$\sum_{t=0}^{|R_i|} z_{ikt} = x_j \qquad \forall k \quad \forall i$$

Locate sensors on different routes in the network so that the Expected variance travel time reduction on the routes is maximized

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$$\max \sum_{i=1}^{I} \sum_{j\in R_i} \sum_{k\in R_i} \sum_{h=0}^{|R_i|} \sum_{t=0}^{|R_i|} \Delta\gamma_i^{jk} \delta_{iht}^{jk} (1-q)^2 q^{h+t}$$

$$\sum_{j=1}^{J} x_j \leq B$$

Locate *B* sensors

$$\sum_{h=0}^{|R_i|} y_{ijh} = x_j \qquad \forall j \quad \forall i$$

$$\sum_{t=0}^{|R_i|} z_{ikt} = x_j \qquad \forall k \quad \forall i$$

Each located sensor is assigned an upstream level and a downstream level

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$$\sum_{j \in R_i} y_{ijh} \leq 1 \qquad \forall i \quad h = 0$$

$$\sum_{j \in R_i} y_{ijh} \leq \sum_{j \in R_i} y_{ij(h-1)} \qquad \forall i \quad h = 1, \ldots, |R_i|$$

$$\sum_{j \in R_i} z_{ijh} \leq \sum_{j \in R_i} y_{ijh} \qquad \forall i \quad h = 0, \ldots, |R_i|$$

For each route R_i :

-all the upstream (downstream) levels start from 0

- each located sensor is assigned exactly one upstream level and one downstream level

- the level assignment forms a consecutive sequence

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$$\delta_{iht}^{jk} \le \frac{1}{2}(y_{ijh} + z_{ikt}) \qquad \forall j,k \quad \forall i \qquad \forall h,t = 0,1,\ldots,|R_i|$$

If *j* is an upstream sensor Site of level *h* and *k* is a downstream sensor Site of level *t* on Route *i* then the corresponding variable delta is assigned value equal to 1

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What we did:

-We have reviewed the problems of optimally locating AVI READERs on a traffic network to monitor travel time performances

- We presented two basic models to optimally locate AVI READERs either to maximize the total vehicle-miles monitored (MODEL AVI3) or the total travel time variance reduction (MODEL AVI4)

-We presented the mathematical formulations of both the models when
- link failures are taken into account
- sensor failures are taken into account

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What's more to be done:

- Experiments with travel times predictions
    - Need data
    - Need to calibrate the distributions

- Extensions of the models when the sensor failure probabilities are site specific

- Development of good heuristics to solve the problems on real case instances

-…

-…

That's all folks !

Questions?